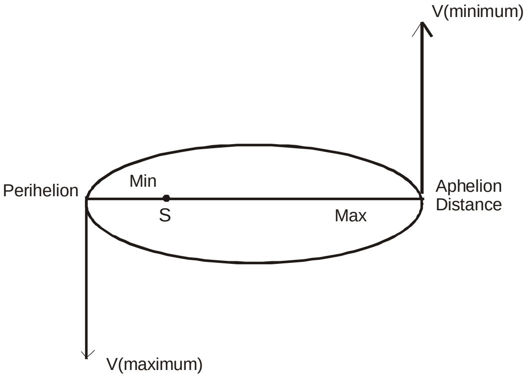
GRAVITATION

SYNOPSIS

Kepler's Laws :

Kepler's Laws of Planetory Motion:

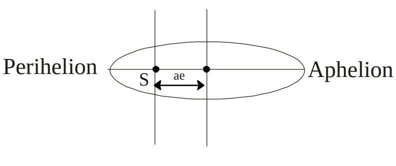
Kepler's first law or laws of orbits: Every planet revolves around the sun in elliptical orbit with the sun is at one of its foci of ellipse. It is also called **" Law of Orbits "**.



Perigee:- The position of a planet nearest to the sun is known as perigee. In this position, the speed of the planet is maximum.

Apogee:- The position of a planet at the maximum distance from the sun is known as apogee. In this position, the speed of the planet is minimum.

➢ A planet of mass 'm' is moving in an elliptical orbit around the sun. The sun, of mass 'M', is at one focus 's' of the ellipse. The other focus is 's', which is located in empty space. Each focus is at distance 'ea' from the ellipse's centre, with 'e' being the eccentricity of the ellipse. The semi major axis of the ellipse 'a', the perihelion distance $r_{\min}$ (nearest from the sun), the aphelion distance $r_{\max}$ (farthest from the sun) are indicated in the figure



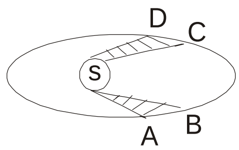
$r_{\min} = a - ea = (1-e)a$

The distance of each focus from the centre of the ellipse is 'ea', where 'e' is the dimensionless number between 0 to 1, called the eccentricity. If e=0, the ellipse is a circle. For earth, e =0.017

Kepler's second law or Laws of Areas: The speed of planet in its orbit varies in such a way that radius vector joining the planet to the sun sweeps out equal areas in equal intervals of time. It also states that radius vector of planet sweeps equal areas in equal intervals of time.

➢ Areal Velocity of radius vector $\left(\frac{dA}{dt}\right)$ joining the planet to sun remains constant. Mathematically

$\frac{dA}{dt} = \text{constant}$



But $A = \frac{1}{2} dl\, r = \frac{1}{2}(r d\theta) r = \frac{1}{2} r^2 d\theta$

So, $\frac{d}{dt}\left(\frac{1}{2} r^2 d\theta\right) = \text{constant}$

$\frac{1}{2} r^2 \frac{d\theta}{dt} = \text{constant}$

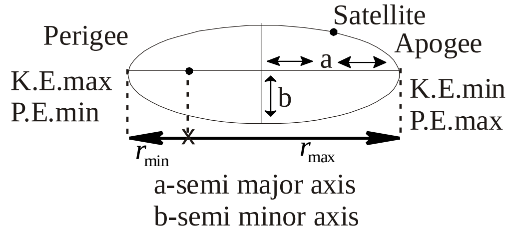
$\frac{1}{2} r^2 \omega = \text{constant}$

$\frac{1}{2}\frac{mr^2\omega}{m} = \frac{I\omega}{2m} = \frac{L}{2m}$

$\frac{L}{2m} = \text{constant},$

L = constant (as the gravitational force on planet by sun is central, so torque is zero and hence angular momentum is constant).

Hence this law is consequence of law of conservation of angular momentum.

➢ 

As angular momentum is conserved so

$$m(V_{\max})(r_{\min}) = m(V_{\min})(r_{\max})$$

$$\Rightarrow \frac{V_{\max}}{V_{\min}} = \frac{r_{\max}}{r_{\min}} = \frac{(1+e)a}{(1+e)a} = \frac{1+e}{1-e}$$

here $V_{perihelion} = V_{\max}$ and $V_{apehelion} = V_{\min}$

$r_{perihelion} = r_{\min}$,

$r_{apehelion} = r_{\max}$

$r_{apehelion} + r_{perihelion} = 2a$, 'a' is the semi major axis.

➢ If e>1 and total energy (K.E +P.E)>0, the path of the satellite is hyperbolic and it escapes from its orbit.

➢ If e<1 and total energy is negative, it moves in an elliptical path.

➢ If e=0 and total energy is negative, it moves in circular path.

➢ If e = 1 and total energy is zero, it will take parabolic path.

➢ The path of the projectiles thrown to lower heights is parabolic and thrown to greater heights is elliptical.

➢ Kepler's laws may be applied to natural and artificial satellites as well.

➢ $$\frac{dA}{dt} = \frac{L}{2m} = \frac{mvr}{2m}$$

$$\frac{dA}{dt} = \frac{Vr}{2}$$

Areal velocity is independent of mass of the satellite.

Kepler's third law : The square of period of revolution of planet around the sun is proportional to cube of the average distance of planet (i.e., semi major axis of elliptical orbit) around the sun.

$$r_{mean} = \frac{r_{\max} + r_{\min}}{2} = \frac{(1+e)a + (1-e)a}{2} = a$$

Hence $T^2 \propto a^3$

where 'a' is length of semi major axis of ellipse

➢ Gravitational Force of Sun provides the necessary centripetal force for the planet to go round the Sun.

If M = mass of Sun

m = mass of planet

r = average distance of the planet from the Sun

then $F = \dfrac{GmM}{r^2} = mr\omega^2$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}\left(as\ \omega = \frac{2\pi}{T}\right)$$

$$T^2 = 4\pi^2 \frac{r^3}{GM} \rightarrow (1)$$

$T^2 \alpha r^3$

Universal Law of Gravitation Basic Forces in Nature:

Depending upon strength and their relative nature, basic forces are classified into four categories

a) Gravitational Force

b) Electro magnetic Force

c) Strong nuclear Force

d) Weak nuclear Force

Relative strengths of basic forces between protons

Basic force	**Range**	**Relative strength**
Gravitational	Long range, infinity	1
Weak nuclear	Extremely short	10^{31}
Electromagnetic	Long range, infinity	10^{36}
Strong nuclear	Short range	10^{38}

Gravitational Force:

➢ This force is responsible for attraction between any two massive particles.

➢ It is always attractive force.

➢ It is a conservative force.

- It is independent of medium present between the masses.
- It is the weakest force.
- It can provide radial acceleration.
- It is a long range force.
- It is communicated through a particle called as **Gravitaion.**

Electro Magnetic Force :

- This force exists between the charges of the atoms and molecules.
- This force is either attractive or repulsive.
- It is also a long range force.
- It is communicated through **Photons**.

Strong Nuclear Force :

- This force may act between a pair of nucleons that is between proton - proton, proton - neutron and neutron - neutron .
- It is charge independent force.
- It is spin dependent force.
- It is a short range force.
- It is communicated through π measons.

Weak Nuclear Force :

- Weak Nuclear Forces are responsible for radioactive decay like β - decay and other similar decays.
- It acts between all leptons, positrons, μ-mesons,neutrinos) and Hadrons, (mesons,baryons)
- It is communicated through weak bosons.

Note:

- Range of gravitational force > Range of electromagnetic force>Range of nuclear force
- Strength of nuclear force> strength of electromagnetic force> strength of gravitational force.

Newton's Law of Gravitation:

Every particle in the universe attracts every other particle with a force.

The force of attraction between two point masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{d^2}.$$

where m_1 and m_2 are masses of two particles, d is the distance of separation between their centres and G is universal gravitational constant

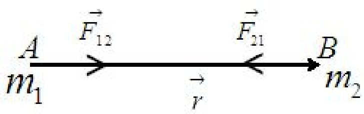
Value of $G = 6.67\times10^{-11}\ Nm^2kg^{-2}$ (or) 6.67×10^{-8} dyne cm^2gm^{-2}

G is a scalar quantity with dimensional formula $M^{-1}L^3T^{-2}$

- Kepler's laws can be deduced from Newton's law of gravitation.
- A mass 'M' is split into two parts and separated by certain distance, the gravitational force between them is maximum only when the two parts are of equal mass i.e., $\frac{M}{2}$ and $\frac{M}{2}$.

Properties of gravitational force:-

- Gravitational force acts along the line joining the two interacting particles ie., gravitational force is a central force.
- Gravitational force between a pair of particles is independent of the medium between the particle and also independent of the presence of other particles.
- Gravitational force between two particles form an action - reaction pair, which are equal in magnitude and opposite in direction. So gravitational force obey's Newtons's III Law



Let $\vec{r}$ be the position vector of B w.r.t A.

$$\vec{F}_{12} = \frac{Gm_1m_2}{r^2}\hat{r} = \frac{Gm_1m_2}{r^3}\vec{r} \text{ (acts along AB)}$$

Here $\hat{r}$ is the unit vector in the direction of $\vec{r}$ and r is the distance between two massive bodies.

$$\vec{F}_{12} = \frac{-Gm_1m_2}{r^2}\hat{r} = \frac{-Gm_1m_2}{r^3}\vec{r} \text{ (acts along BA)}$$

$$\Rightarrow \overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

$$\Rightarrow \overrightarrow{F_{12}} + \overrightarrow{F_{21}} = 0$$

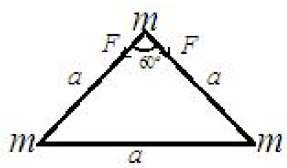
- Gravitational force is a conservative force i.e., the amount of work done by gravitational force in displacing a body from one place to another is independent of the path traversed.
- It there are more than two interacting bodies, the resultant gravitational force on any one body can be obtained by the "super position principle". According to it the net gravitational force on any particle is the vector sum of individual gravitational forces on it by all other particles in the system.

$$\overrightarrow{F_{1,net}} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \overrightarrow{F_{14}} + ... + \overrightarrow{F_{1n}}$$

we can express this equation more compactly as

vector sum $\overrightarrow{F_{1,net}} = \sum_{i=2}^{n} \overrightarrow{F_{1i}}$

- If two particles of equal mass 'm' are placed at the two vertices of an equilateral triangle of side 'a', then the resultant gravitational force on mass 'm' placed at the third vertex is given by

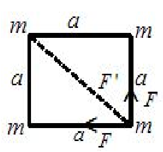


$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} \quad (\theta = 60^0)$$

$$= \sqrt{3}F \quad [\because F_1 = F_2 = F]$$

$$F_R = \sqrt{3}\left[\frac{Gm^2}{a^2}\right]$$

- If equal masses each of mass m, are kept at the vertices of a square of length a, the gravitational force of attraction on any one of the particles is



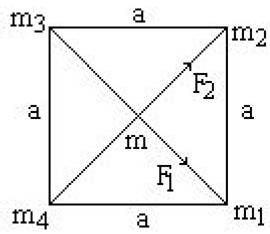
$$F_{net} = \sqrt{2}F + F' = \frac{\sqrt{2}Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$F_R = \frac{Gm^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$ along the diagonal towards the opposite corner.

- If four different masses m_1, m_2, m_3 and m_4 are placed at the four corners of a square of side a the resultant gravitational force on mass m kept at the centre.

The force on m due to m_1 and m_3 is

$$F_1 = \frac{2Gm}{a^2}(m_1 - m_3) \text{ [if } m_1 > m_3]$$



along the diagonal towards m_1.

The force on m due m_2 and m_4 is

$F_2 = \frac{2Gm}{a^2}(m_2 - m_4)$ [if $m_2 > m_4$] ; along the diagonal towards m_2.

The resultant force is $\sqrt{F_1^2 + F_2^2} = F$.

$$F = \frac{2Gm}{a^2}\sqrt{(m_1 - m_3)^2 + (m_2 - m_4)^2}$$

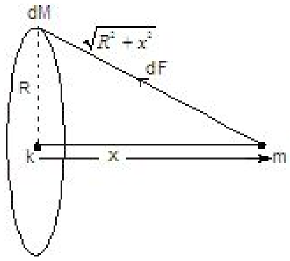
The resulatant force makes an angle θ with F_1 and $\theta = \tan^{-1}(\frac{F_2}{F_1})$.

- A thin rod of mass M and length L is bent in a semi circle.gravitational force on a particle with mass"m" is placed at center of curvature is

$$F = \frac{GMm}{L^2}2\pi$$

- A thin rod of mass M and length L is bent into a complete circle,then resultant force on it is zero
- A point mass m is at a distance 'x' from the center of the ring of mass M and radius R on its axis.gravitational force between two is

$$F = \frac{GMmx}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$$



- If x>>>R $\Rightarrow F = \frac{GMm}{x^2}$, then for a distant point, ring behaves as point mass.
- If x<<<R $\Rightarrow F = \frac{GMm}{R^3}x$,then force varies linearly as distance 'x'
- Force is maximum, at x= $\pm\frac{R}{\sqrt{2}}$, maximum force $F_{\max imum} = \frac{2GMm}{3\sqrt{3}R^2}$
- Force exerted on moon by sun is greater than exerted by earth.despite moon does not escape out from earth at the time of solar eclipse because the gravitational pull of sun provides necessary centripetal force for orbital motion.
- Sun exerts gravitational force on earth ,but earth does not move towards sun because the gravittional pull of sun provides necessary centripetal force to earth.so that the orbit is stable one.

Variation of 'g' :

- Line joining the places on the earth having same values of g are called isograms. Gravity meters, Etvos gravity balances are used to measure changes in acceleration due to gravity

Variation of g with altitude : If g and g_h are acceleration due to gravities on the surface of the earth and height 'h' above the surface of the earth of mass M and radius R then

$$g = \frac{GM}{R^2} \text{ and } g_h = \frac{GM}{(R+h)^2}$$

- For small values of h, $g_h = g\left(1-\frac{2h}{R}\right)$

thus as the height increases, the value of g decreases.

- The decrease in value of g at height $h(h << R)$ is

$$\Delta g = g - g_h = \left(\frac{2h}{R}\right)g$$

- The fractional change in value of g is $\frac{\Delta g}{g} = \frac{2h}{R}$

Variation of g with depth: If g is acceleration due to gravity at the surface of the earth and g_h is acceleration due to gravity at a depth d below the surface of the earth, then

- on surface, g = $\frac{4}{3}\pi GR\rho$
- at a depth, $g_d = \frac{4}{3}\pi G(R-d)\rho$

where r = R – d

- $g_d = g\left(1-\frac{d}{R}\right)$

thus as the depth d increases, the acceleration due to gravity decreases.

- The decrease in value of g at depth d is $\Delta g = gd/R$
- Decrease in g at small heights is more than decrease in g at small depths.
- But decrease in g at large heights is less than decrease in g at large depths.

Variation of g due to rotation of earth : Due to the rotation of earth, the value of acceleration due to gravity g_λ at a given place is given by $g_\lambda = g - r\omega^2$

where r is radius of the body revolving in a circle where r = R . cos λ

$$g_\lambda = g - \omega^2 R\cos^2\lambda$$

where ω is the angular velocity. R is radius of the earth and λ is latitude of the place

Special cases :

- At the poles $\lambda = 90^0$

$$\therefore g_{\lambda(=90^o)} = g - \omega^2 R(0)^2 \quad (\cos 90 = 0)$$

$\therefore g_{\lambda(=90^o)} = g$

Hence the value of g at poles does not change due to rotation of earth.

The value of g is maximum at poles

- At the equator $\lambda = 0^0$

$\therefore g_{\lambda(=0^o)} = g - \omega^2 R(1)^2 \quad (\cos 0 = 1)$

$\therefore g_{\lambda(=0^o)} = g - \omega^2 R$

Hence the value of g at equator changes much due to rotation of earth.

The value of g is minimum at equator.

- The time period of earth when earth rotates 17 times faster, so that $g' = 0$ at equator

$$T = 2\pi\sqrt{\frac{R}{g}} = 84.6\min = 5076s = 1.41hr$$

Note:-

- The value of acceleration due to gravity 'g' at poles does not depend on the speed of rotation of the earth. But at the equator, the acceleration due to gravity 'g' decreases with the increase of speed of rotation of earth.
- If earth suddenly stops its rotation, then the acceleration due to gravity at poles remains constant. And also acceleration due to gravity at equator increases by $\omega^2 R$.
- If the angular speed of earth becomes 17 times its present value then the value of acceleration due to gravity at the equator becomes zero.

Variation of g due to shape of the earth:

The earth is elliptical in shape, it is flattended at the poles and bulged out at the equator so radius at equator is greater than the radius at poles and as $g \alpha \frac{1}{R^2}$, there fore the value of g at the equator is minimum and at the poles is maximum

Gravitational Field:

- The concept of gravitational field is used to overcome the difficulties encountered in universal law of gravitation.
- Einstein considered gravitational field as a distortion of 'space' due to the presence of matter..

Gravitational field strength (or) Intensity of Gravitationl Field:

- Gravitational field strength at any point in a gravitational field is defined as the gravitational force experienced by a unit mass placed at that point.

$\therefore$ Gravitational field strength,

$$\overrightarrow{E_g} = \frac{\vec{F}}{m_0}$$

Units of gravitational field strength are Nkg^{-1} or ms^{-2} and dimensional formula is LT^{-2}

It is a vector quantity. It is always directed radially towards the centre of mass of the body producing the field.

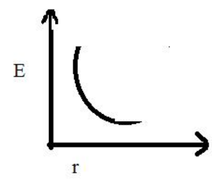
Note: In the earth's gravitational field,

$$\overrightarrow{E_g} = \frac{\vec{F}}{m_0} = \frac{m_0\vec{g}}{m_0} = \vec{g}.$$

Hence in the earth's gravitational field, the intensity of gravitational field is equal to acceleration due to gravity 'g'.

- The intensity of gravitational field at a distance r from a point mass 'M' is given by

$$E_G = \frac{GM}{r^2}$$



In vector form the above formula is $\overrightarrow{E_G} = \frac{GM}{r^3}\overrightarrow{(r)}$.

Theoritically gravitational field due to a particle extends upto infinite distance arround it

The value of E_g is zero at $r = \alpha$.

If the system has a number of masses, then resultant gravitational field intensity can be found out by using the principle of super-position.

i.e. $\overrightarrow{E_g} = \overrightarrow{E_{g_1}} + \overrightarrow{E_{g_2}} + \overrightarrow{E_{g_3}} + ...$

Note: Gravitational field posses energy and momentum

Propagation of gravitational field:

- According to Einstein's general theory of relativity, whenever a body with mass is accelerated, the gravitational field around it undergoes rapid changes.
- Just as photon in electromagnetic field, a quantum of energy is associated with gravitational field called 'graviton'.
- gravitons, like photons are mass less, electrically uncharged particles, assumed to travell at the speed of light and would be emitted by highly accelerating and extremely massive objects such as stars.
- The point between two massive objects at which fields are equal in magnitude but opposite in direction is called null point.

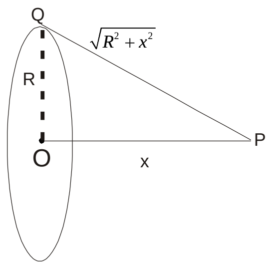
If two particles of masses $m_1 \& m_2$ are separated by a distance 'd', the distance of null point from m_1 is given by

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1}$$

Field due to Circular Ring:

- Gravitational field intensity due to a circular ring at any point on its axis is

$$E_g = \frac{GMx}{\left(X^2 + R^2\right)^{3/2}} \text{ along } \overrightarrow{PO}$$



Gravitational field intensity is directed towards the centre of the circular ring.

At the centre of the circular ring, $E_g = 0$

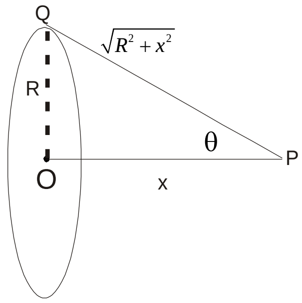
E_g is maximum , at $x = \frac{R}{\sqrt{2}}$ and

$$E_{max} = \frac{2GM}{3\sqrt{3}R^2}$$

Field due to Circular Disc:

- Gravitational field intensity due to a circular disc at any point on the axial line

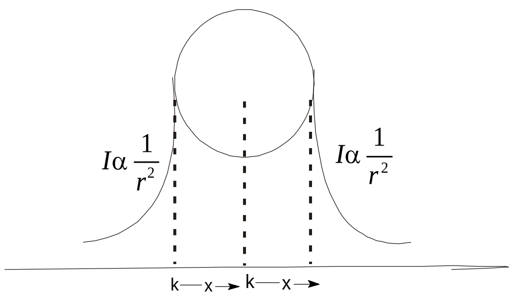
$$E_g = \frac{2GM}{R^2}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$



or $E_g = \frac{2GM}{R^2}(1 - Cos\theta)$ (in terms of 'θ')

Field due to hallow sphere (or) Field due to Sperical Shell:

- Gravitational field intensity due to a thin uniform spherical shell



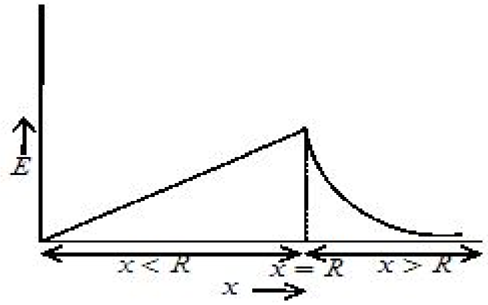
$$\left(E_g\right)_{center} = zero$$

$$\left(E_g\right)_{surface} = \frac{GM}{R^2}$$

At a point inside the spherical shell, $(E_g)_{inside} = 0$

At a point outside the spherical shell, $(E_g)_{outside} = \frac{GM}{x^2}$ (here x>R)

Field due to Solid Sphere(uniform mass density): Gravitational field intensity due to a solid



$$E_{g\ inside} = \frac{GMx}{R^3}\ (\ for\ x<R)$$

At a point on the surface of the solid sphere,

$$E_{g_{surface}} = \frac{GM}{R^2}\ (\ for\ x=R)$$

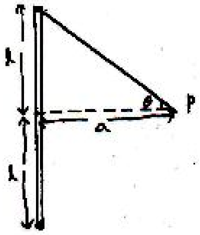
At a point outside the solid sphere,

$$(E_g)_{outside} = \frac{GM}{X^2}\quad (\ for\ x>R)$$

$E_g = 0$ (at infinite distance)

$E_g = 0$ (at the centre of solid sphere)

Field due to Straight Rod: Intensity due to rod of length 2l,density ρ ,placed along x- axis,such that mid point of rod is coincides with origin.The gravitational field intensity at a point P(a,o) is



$$E_g = \frac{2G\rho}{a}\frac{l}{\sqrt{l^2+a^2}}$$

above expression can be written as

$$E_g = \frac{2G\rho}{a}\frac{1}{\sqrt{1+\frac{a^2}{l^2}}}$$

$$E_g = \frac{2G\rho}{a}\left(1-\frac{a^2}{2l^2}+high\ powers\ of\ \frac{a^2}{l^2}+-----\right)$$

if l is ∞ then $E_g = \frac{2G\rho}{a}$

Gravitational Potential:

- The amount of work done in bringing a unit mass from infinity to a certain point in the gravitational field of another massive object is called as gravitational potential.

This work is obtained by the agent in bringing the mass. Let W joule of work is obtained in bringing a test mass m_0 it from infinity to some point, then gravitational potential at that point will be

$$V = \frac{W}{m_0}$$

Since, this work is negative. Hence, gravitational potential is always negative.

unit $\rightarrow$ J/Kg (S.I. System)

Dimensional formula $\rightarrow \left[M^0L^2T^{-2}\right]$

Potential due to a point mass:

- The gravitational potential at a point p which is at a distance r from a point mass M is given by

$$V = -\frac{GM}{r}$$

- For an assembly of number of masses m_1 , m_2 , m_3 .... m_n at distances r_1 , r_2 , r_3 .... r_n from the point 'p', the resultant gravitational potential at a point 'p' can be written as

$$V = V_1+V_2+V_3+.........V_n$$

$$\Rightarrow V = -G\left[\frac{m_1}{r_1}+\frac{m_2}{r_2}+\frac{m_3}{r_3}+..........+\frac{m_n}{r_n}\right]$$

$$\Rightarrow V = -G\sum_{i=1}^{n}\frac{m_i}{r_i}$$

Potential due to Circular Ring:

➢ Gravitational potential due to a circular ring, at a distance r from the centre and on the axis of a ring of mass M and radius x is given by

$$V = \frac{-GM}{\sqrt{R^2 + x^2}}$$

Gravitational potential due to a spherical shell:

Let M be the mass of spherical shell and R is its radius

At a point inside the spherical shell,

$$V_{inside} = \frac{-GM}{R} \text{ (If x<R)}$$

At a point on the surface of the spherical shell,

$$V_{surface} = \frac{-GM}{R} \text{ (If x=R)}$$

$$v_{centre} = -\frac{GM}{R} \text{ (r=0 at centre)}$$

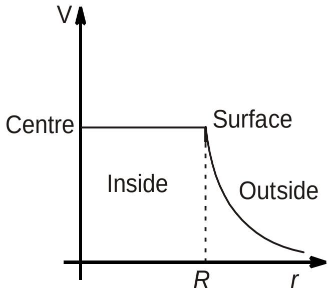
$$v_{inside} = v_{surface} = v_{centre} = -\frac{GM}{R},$$

At a point outside the spherical shell,

$$V_{outside} = \frac{-GM}{x} \text{ (If x>R)}$$

At infinity, $V_{\infty} = 0$

The variation of magnitude of V with r is as shown



Gravitational potential due to a solid sphere:

At a point inside the solid sphere,

$$V_{inside} = \frac{-GM}{2R^3}(3R^2 - x^2)$$

$$V_{inside} = -GM\left(\frac{3}{2R} - \frac{r^2}{2R^3}\right) \text{(if x<R)}$$

At a point on the surface of the solid sphere,

$$V_{surface} = \frac{-GM}{R} \text{ (If x=R)}$$

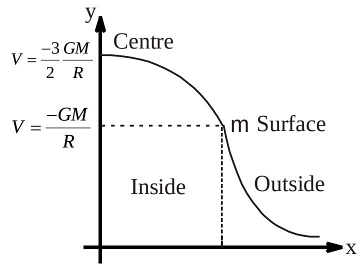
At a point outside the solid sphere,

$$V_{outside} = \frac{-GM}{x} \text{ (If x>R)}$$

At the centre, x=0

$$\Rightarrow V_c = -\frac{3}{2}\frac{GM}{R} = \frac{3}{2}V_{surface}.$$

The variation of V with x is as shown:



In case of solid sphere potential is maximum at centre.

Newtons Shell Theorem : Gravitational potential at a point outside of a solid (or) hallow sphere of mass M is same as potential at that point due to a point mass of M seperated by same distance hence the sphere can be replaced by a point mass.

Relation Between Gravitational Field and Potential:

Gravitational field and the gravitational potential are related by the following relation.

$$\vec{E} = -\text{gradient}V$$

$$= -\text{grad}V$$

$$= -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\frac{\partial V}{\partial x}$ = Partial derivative of potential function V with respect to x, i.e., differentiate V wrt x assuming y and z to be constant.

The above equation can be written in the following forms.

- $E = \frac{-dV}{dx}$, If gravitational field is along x-direction only.
- $dV = -\vec{E}.d\vec{r}$, (where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$ and $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$)

Gravitational Potential Energy:

The amount of work done against the gravitational force in bringing a body from infinity to any point in the gravitational field is defined as the gravitational potential energy at that point.

For a conservative field, $F = -\frac{dU}{dr}$

$$dU = -\vec{F}.d\vec{r}$$

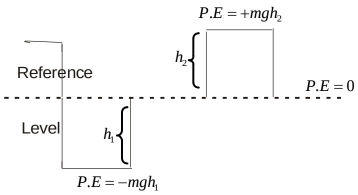
$$\Rightarrow \int_{u_0}^{u} dU = -\int_{r_0}^{r} \vec{F}.d\vec{r} \Rightarrow U - U_0 = -\int_{r_0}^{r} \vec{F}.d\vec{r}$$

We generally choose the reference point at infinity and assume potential energy to be zero there. If we take $r_0 = \infty$ and $U_0 = 0$ then we can write

$$\Rightarrow U = -\int_{\infty}^{r} \vec{F}.d\vec{r} = -W\left[as\int_{\infty}^{r} \vec{F}.d\vec{r} = W\right]$$

Potential energy of a body or system is negative of work done by the conservative forces in bringing it from infinity to present position.

- In case of conservative field, potential energy is equal to -ve of work done in shifting body from some reference point to given position.



- Particle moves opposite to field, work done by field will be –ve. So change in potential energy will +ve. So potential energy will increase.
- Particle moves along the field ,work done is +ve,so change in potential energy is –ve, potential energy decreases.
- potential energy exists for only conservative force,it does not exist for non conservative
- Potential energy depends on frame of reference.+ve potential energy means body will do work in returning to its referenceposition.-ve potential energy means workdone on body to bring to back to reference position.

Now by the definition of gravitational potential, we can write

$$V = -\frac{W}{m} = \frac{U}{m} \Rightarrow U = mV$$

- Self gravitational potential energy of a planet $-3\frac{GM^2}{2R}$

Gravitational Potential Energy of a Two Particle System:

- The gravitational potential energy of two particles of masses m_1 and m_2 separated by a distance r is given by

$$U = -\frac{Gm_1m_2}{r}$$

Gravitational Potential Energy For a System of Particles:

- The gravitational potential energy for a system of particles is given by

$$U = \sum U_i = -\left[\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + ...\right]$$

$$= -G\left[\frac{m_1m_2}{r_{12}} + \frac{m_2m_3}{r_{23}} + ...\right]$$

For a n particle system there are $\frac{n(n-1)}{2}$ pairs and the potential energy is calculated for each pair and added to get the total potential energy of the system.

Gravitational Potential Energy of a Body in Earth's Gravitational Field:

- If a point mass 'm' is at a distance r from the centre of the earth.

$$U = -\frac{GMm}{r}$$

At the surface of earth, $U_{surface} = -\frac{GMm}{R} = -mgR\left(\because g = \frac{GM}{R^2}\right)$

At a height 'h' above the surface of earth, $U_h = -\frac{GMm}{R+h}$

The difference in potential energy of the body of mass m at a height h and on the surface of earth is $\Delta U = U_h - U_{on}$

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{r}\right) = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$= \frac{GMmh}{(R+h)R} = \frac{GMmh}{R^2\left(1+\frac{h}{R}\right)}$$

$$\Delta U = \frac{mgh}{1+\frac{h}{R}}$$

If $h << R$, $\Delta U \approx mgh$

Note :

- Work done in lifting a body of mass m from earth surface to a height h above the earths surface is

$$W = U_h - U_{on}$$

$$W = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$W = \frac{mgh}{1+\frac{h}{R}}$$

- Gravitational potential energy at the centre of the earth relative to infinity is given by

$$U_c = mv_c = -\frac{3}{2}\frac{GMm}{R}$$

here $V_c = \frac{3}{2}V_s = \frac{-3GM}{2R}$ (It is minimum but not zero, however 'g' at centre of earth is zero)

Ecape Velocity:

- The minimum velocity given to a body in order to throw it out of the gravitational field of a planet (earth) is defined as escape velocity.
  For a body to just escape it is projected with a minimum velocity such that its total energy must be zero.

If V_e is the escape velocity from the surface of the planet then $PE + KE = 0$

$$-\frac{GMm}{R} + \frac{1}{2}mV_e^2 = 0$$

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R}$$

$V_e = \sqrt{\frac{2GM}{R}}$. Also $V_e = \sqrt{2gR}$ and

$V_e = \sqrt{2\left(\frac{4}{3}\pi R\rho G\right)R}$ (Where ρ is the mean density of the planet, M is the mass of planet and R is the radius of planet).

Escape Velocity of a body From certain Height above the surface of a planet:

- Consider a body of mass 'm' placed at rest at a height 'h' above the surface of a planet of mass M and radius R then gravitational potential energy of the body is

$$PE = \frac{-GMm}{R+h}.$$

Minimum kinetic energy imparted to the body to escape it from the planet is

$$KE = -PE = \frac{GMm}{R+h} \Rightarrow \frac{1}{2}mV_e^2 = \frac{GMm}{R+h}$$

$$\Rightarrow Ve = \sqrt{\frac{2GM}{R+h}} = \sqrt{2g_h(R+h)}$$

Where g_h = acceleration due to gravity at height h.

Behaviour of a Body Projected Vertically Up with Different Velocities from the Surface of a Planet:

- Consider a body of mass 'm' projected with a velocity 'V' from the surface of a planet of mass 'M' and radius 'R'

Case I: If the velocity of projection $V < V_e$ then

Total energy is negative

The body goes to certain maximum height and then falls.

To find this maximum height we use law of conservation of energy

$$T.E_{\text{surface}} = T.E_{\text{max.height}}$$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h}$$

$$\frac{1}{2}mv^2 = \frac{GMmh}{R(R+h)}, \quad \frac{R+h}{h} = \frac{2GM}{RV^2}$$

$$\frac{R}{h} + 1 = \frac{V_e^2}{V^2}, \quad \frac{R}{h} = \left(\frac{V_e^2}{V^2} - 1\right)$$

$$h = \frac{R}{\left(\frac{V_e^2}{V^2} - 1\right)}$$

Case II:-If the velocity of projection $V = V_e$ then total energy of the body just becomes zero so that the body just escapes

from the planet and goes to infinity and the body possess zero velocity at infinity

case III:- If a body is projected with a velocity greater than the escape velocity $(V > V_e)$

Total energy is positive,

The body escapes from gravitational influence of a planet and enter into interstellar spacewith certain velocity.

By conservation of energy

$$\frac{1}{2}mV^2 - \frac{GMm}{R} = \frac{1}{2}mV_1^2$$

$$\Rightarrow V_1^2 = V^2 - \frac{2GM}{R} = V^2 - V_e^2 \left[\because V_e^2 = \frac{2GM}{R}\right]$$

$$\Rightarrow V_1 = \sqrt{V^2 - V_e^2}$$

i.e., the body will move in interplanetary or interstellar space with a velocity $\sqrt{V^2 - V_e^2}$

Salient features regarding escape velocity:

- Escape velocity depends on the mass, density and radius of the planet from which the body is projected.
- Escape velocity does not depend on the mass of the body, its direction of projection and the angle of projection.
- Greater the value of escape velocity from a planet, denser will be its atmosphere.
- Escape velocity on the surface of earth = 11.2 Km/s

  Escape velocity on the surface of moon = 2.31 Km/s

  Escape velocity on the surface of Jupiter = 42 Km/s

  Escape velocity from the solar system= 641 Km/s
- There is no atmosphere on moon, because r.m.s. velocities of molecules are greater than the escape velocity $(i.e., V_{rms} > V_e)$
- The escape velocity on sun is the maximum. As $V_{rms} < V_e$, hence even the lightest molecules cannot escape from there. Sufficient amount of hydrogen is present in the atmosphere of the sun since the escape velocity on the sun is very high. There is no atmosphere on the surface of the moon since the escape velocity on the surface of the moon is less.
- If a body falls freely from infinity, then it reaches the earth with a velocity of 11.2 Km/s.
- If the velocity of the satellite orbitting near the surface of earth is increased by 41.4% (or) increased to $\sqrt{2}$ times, it escapes out from the earth because its velocity becomes equal to the escape velocity.

Earth Satellites

Satellities: The bodies revolving round a planet in its gravitational field are defined as satellites. Different types of satellites are

Natural Satellites: The natural bodies which revolve around a planet are known as natural satellites. For example, moon is the natural satellite of earth. There are12 natural satellites of Jupiter,10 of Saturn and 2 of Mars.

Artificial Satellites: The man made bodies which are established in a particular orbit and are made to revolve around the earth are known as artificial satellites. For example Aryabhatta, Rohini, Bhaskara etc are artificial satellites.

Orbital speed of Satellites: The velocity of a satellite revolving around earth of mass M and radius R in a circular orbit of radius 'r' at a height 'h' from the surface of earth is called orbital velocity.

$$V_o = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

$$\omega = \sqrt{\frac{GM}{(R+h)^3}}$$

For a satellite orbiting very close to earth.

$h << R$ then, $V_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$\omega^2 \propto \frac{1}{R^3} \Rightarrow T^2 \propto R^3$$

For two satellites revolving around the earth in different cicular orbits of radii r_1 and r_2 at vertical heights h_1 and h_2; $\frac{V_1}{V_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{R+h_2}{R+h_1}}$

Orbital velocity for a satellite close to the surface of earth $V_0 = 7.92\ kms^{-1} \approx 8kms^{-1}$

Orbital velocity is independent of mass of the satellite. It is always along the tangent to the orbit.

Relation between escape and orbital velocities

$V_e = \sqrt{2}\ V_0$

If the speed of the orbitting body is made $\sqrt{2}$ times its inital velocity or velocity is increased by 41.4% or its KE is doubled (increased by 100%), then the body will escape.

If the speed of the orbiting body 'V' is such that $V_o < V < V_e$ then its orbit changes from circle to ellipse

Time Period of Revolution: Time taken by the satellite in completing one revolution round the earth is called as its time period (T).Time period of revolution,

$$T = \frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$

$$= \frac{2\pi r}{\sqrt{\frac{GM}{r}}} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\left[\because g = \frac{GM}{R^2}\right]$$

If the satellite revolves close to the earth surface, h<<R $\Rightarrow$ Time period of revolution,

$T = 2\pi\sqrt{\frac{R}{g}} = 84.6\,\text{min}$ =1.41 hr and

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

This is same as timeperiod of a simple pendulum of infinite length and time period of SHM of a ball in a tunnel through the earth.

Frequency of Revolution(n): The number of revolutions made by the satellite in one second is called the frequency of revolution(n).

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{GM}{r^3}} = \frac{1}{2\pi}\sqrt{\frac{GM}{(R+h)^3}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{gR^2}{(R+h)^3}}\left[\because g = \frac{GM}{R^2}\right]$$

If the satellite revolves close to the earth surface,

h<<R $\Rightarrow n=\frac{1}{2\pi}\sqrt{\frac{GM}{r^3}} \Rightarrow \boxed{n=\frac{1}{2\pi}\sqrt{\frac{g}{R}}}$

When the satellite revolves in an orbit of radius 'r' (here r=R+h) potential energy is negative. Here negative energy means that the satellite is moving in the gravitational field of the planet. The planet and the satellite form a bound system. If it is to be escaped out of the gravitational field, then additional energy $\frac{GMm}{2(R+h)}$ will have to be given to it.

Angular Momentum: In case of the satellite motion, the angular momentum of the satellite is given by

$$L=mv_0r=mr\sqrt{\frac{GM}{r}}=m\sqrt{GMr}$$

$$\Rightarrow L=\sqrt{GMm^2r}$$

- As the force is central, so for satellites τ =0, then L=constant in a given orbit.

  Angular momentum of the satellite depends on mass of the satellite, mass of the planet and radius of the orbit.
- A satellite behaves like a freely falling body i.e., only when 'g' is effective on it.
- A body inside a satellite is in the state of weight less ness.
- A satellite can revolve only in that orbit which contains the centre of earth as well as the equator.

  The velocity of a satellite near the planet is maximum, where as its time period is minimum.

  When a satellite revolves round a planet in an elliptical orbit, then its orbital speed is not uniform.
- The mass of a planet can be determined with the help of its satellite.

Note:

- An artificial satellite in the presence of frictional forces will move into an orbit closer to earth and may have increased kinetic energy. Energy of the satellite decreases due to dissipative forces due to which,satellite shifts to lower orbits close to earth and in the process gains kinetic energy.
- If the satellite is travelling in the same direction as the rotation of earth i.e west to east , the time intervel between two successive times at which it will appear vertically overhead to an observer at a fixed point on the equator is $T=\frac{T_sT_e}{T_e-T_s}$

$$\left\{Since\ \omega_{rel}=\omega_{sat}-\omega_{earth} \Rightarrow \frac{2\pi}{T}=\frac{2\pi}{T_s}-\frac{2\pi}{T_e}\right\}$$

- Satellites are non inertial frames and as such pseudo forces (centrifugal force) acts on the astronaut in a direction opposite to the centre of the earth.

Energy of Orbitting Satellite:

- The potential energy of the system is

$$U=\frac{-GMm}{r}$$

The kinetic energy of the satellite is,

$$K=\frac{1}{2}mV_0^2=\frac{1}{2}m\left(\frac{GM}{r}\right)$$

or $K=\frac{GMm}{2r}$

The total energy is $E=K+U=-\frac{GMm}{2r}$

$$E=-\frac{GMm}{2r}$$

Binding Energy: Total energy of a satellite in its orbit is negative. Negative energy menas that the satellite is bound to the central body by an attractive force and energy must be supplied to remove it from the orbit to infinity.

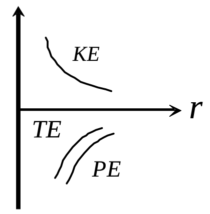
- The energy required to remove the satellite from its orbit to infinity is called binding energy of the system, i.e.,

Binding energy(BE)= $-E = \dfrac{GMm}{2r}$

For a satellite $\dfrac{PE}{KE} = -2$ $\dfrac{PE}{TE} = 2$

$\dfrac{KE}{TE} = -1$, $PE : KE : TE = -2 : 1 : -1$

energy graph for a satellite is



Change in Orbit of a Satellite:

- Energy required to shift a satellite from an orbit of radius r_1 into an orbit of radius r_2 is

$$E = E_2 - E_1$$

$$E = \frac{GMm}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$E = \frac{gR^2m}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

When the satellite is transferred to a higher orbit $(r_2 > r_1)$ then variation in different qunatities can be shown by the following table.

Quantities	**Variation**	**Relation with r**
Orbital Velocity	Decreases	$v\alpha \dfrac{1}{\sqrt{r}}$
Time period	Increases	$T\alpha r^{3/2}$
Linear momentum	Decreases	$p\alpha \dfrac{1}{\sqrt{r}}$
Angular momentum	Increases	$L\alpha \sqrt{r}$
Kinetic energy	Decreases	$K\alpha \dfrac{1}{r}$
Potential energy	Increases	$U\alpha -\dfrac{1}{r}$
Total energy	Increases	$E\alpha -\dfrac{1}{r}$
Binding energy	Decreases	$BE\alpha \dfrac{1}{r}$

Energy Distribution in Eliptical Orbit: If the orbit of satellite is eliptic then the total energy $E = -\dfrac{GMm}{2a}$ = constant (Where a is semi major axis)

- Kinetic energy K will be maximum when the satellite is closest to the central body (at perigee) and minimum when it is farthest from the central body (at apogee).

Potential energy U will be minimum when kinetic energy is maximum i.e., the satellite is closest to the central body (at perigee) and maximum when kinetic energy is minimum i.e., the satellite is farthest from the central body (at apogee).

Important Features Regarding Satellite:

- Total energy of a closed system is always negative. For example, energy of planet - sun, satellite-earth or electron-nucleus system are always negative.

If the law of force obeys the inverse square law

$F\alpha \dfrac{1}{r^2}$, $F = -\dfrac{dU}{dr}$ and $K.E = \dfrac{|U|}{2} = |E|$

U $\rightarrow$ potential energy, E $\rightarrow$ total energy

The same is true for electron-nucleus system because there also, the electrostatic force,

$F_e\alpha \dfrac{1}{r^2}$

In case of satellite motion,

L=mvr=constant $\Rightarrow v\alpha \dfrac{1}{r}$.

According to the equation for the orbital velocity,

$$v_0 = \sqrt{\frac{GM}{r}} \Rightarrow v_0 \alpha \frac{1}{\sqrt{r}}$$

These two results appear to be contradictory. However, this apparent contradiction is resolved if we keep in mind that $v\alpha\frac{1}{r}$ holds good for different points of the orbit while $v\alpha\frac{1}{\sqrt{r}}$ for different orbit as show in the figures

fig (i)

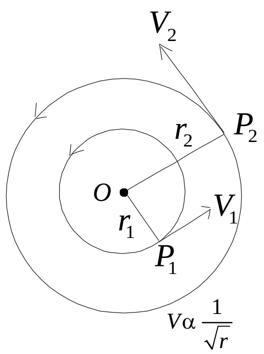
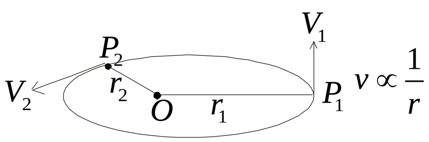


fig (ii)



- A satellite of mass 'm' and radius 'R' is orbitting earth in a circular orbit of radius 'r'. It starts losing energy due to air resistance at a rate of CJs^{-1}. The time taken by the satellite to reach earth is

$$\frac{GMm}{2C}\left[\frac{1}{R}-\frac{1}{r}\right] \left[\because E = \frac{-GMm}{2r}\right.$$

$$\Rightarrow \frac{dE}{dt} = \frac{GMm}{2}\frac{1}{r^2}\frac{dr}{dt} = C$$

$$\Rightarrow dt = \frac{GMm}{2C}\frac{dr}{r^2}$$

$$\int_0^t dt = \frac{GMm}{2C}\int_r^R \frac{1}{r^2}dr \Rightarrow t = \frac{GMm}{2C}\left[\frac{1}{R}-\frac{1}{r}\right]$$

Geostationary and Polar Satellites

Communication Satellites:

- The satellites which remain stationary with respect to earth are known as communication satellites. For example INSAT 1A, 1B, 2A etc are communication satellites.
  The angular velocity of the satellite must be in the same direction as that of earth i.e., from west towards east.
- The period of revolution of satellite must be equal to the period of rotation of earth about its own axis i.e., 24 hrs.
- The orbit of the satellite should be circular and in the equatorial plane of the earth.
- The satellite should be established at a height of 36000 kms from the earth's surface.
- Satellites are in general in the gravitational field of the planet.
- The astronaut sitting in a geo-stationary satellites is in the state of weight lessness as the value of 'g' inside a satellite is zero.
- To determine the time in a geo-stationary satellite, a spring watch is used instead of a pendulum clock because pendulum clock does not oscillate due to 'g' being zero.
- If anything is gently released from a geo-stationary satellite, then it starts moving with the velocity of the satellite and itself becomes a satellite.
- The geo-stationary satellite is projected from west towards east so that maximum benefit of the motion of the earth may be obtained.
- A satellite moving in a stable orbit does not need any energy from an external source.
- Orbits of geostationnary satellites called parking orbits. (h = 36000km, r = 42000km)
- The time period of revolution of the satellite depends upon its height above the earth surface. Higher the height of the satellite, greater will be its time period.
- Time period of revolution of a geostationary satellite is 24 hours.

Conditions for geo-stationary satellite:

- The plane of obit of the satellite should coincide with geo-equatorial plane
- The direction of revolution of satellite should be same as the direction of rotation of earth (ie., from West to East)
- The time period of revolution of the satellite should be 24 hr
- Time period of revolution of geo-stationary satellite with respect to earth is infinity.
- When a satellite is revolving around the earth in a fixed orbit then the astronant inside the satellite experiences weightlessness or zero apparent weight because zero normal reaction is exerted by the satellite on the astronaut and both have equal acceleration towards the centre of the earth.
- It is also known as communication satellite or synchronous satellite whose time period is

$$T^2 = \frac{4\pi^2}{GM}(r^3)$$

$$\Rightarrow r^3 \frac{GMT^2}{4\pi^2}$$

$$\Rightarrow r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \Rightarrow r = \left(\frac{GM}{R^2} X \frac{R^2T^2}{4\pi^2}\right)^{1/3}$$

$$\Rightarrow r = \left(9.8X\frac{(6.4X10^6)(86400)^2}{4X(3.14)^2}\right)^{1/3}$$

$\Rightarrow r = 42400X10^3 m = 42400 Km$

$\Rightarrow$ R+h=42400 Km

$\Rightarrow$ 6400+h=42400 $\Rightarrow$ h=36000 Km

- From above, the height of the geo-stationary satellite from the surface of the earth is nearly 36000 KM.
- The orbital velocity of this satellite is nearly 3.08 Km/s
- The relative velocity of geo-stationary satellite with respect to earth is zero.

  The orbit of the geo-stationary satellite is called the ‘Parking Orbit”.

Polar Satellites:

- These are low altitude (500 km to 800 km) satellites
- They go round the poles of earth in north-south direction
- Polar satellites have a time period of 100 minutes nearly
- These satellites can view polar and equatorial regions at close distances with good resolution.
- These satellites are useful for remote sensing, meteorology and environmental studies of earth.

Weightlessness:

- Weightlessness is a phenomenon in which the object is in a state of free fall
- Pictures of astronauts floating in a satellite illustrate this phenonmenon of weightlessness
- For a satellite revolving around earth, the contents of the satellite (including the satellite) experience accelaration towards the centre of earth, which equals earth’s acceleration due to gravity at that position
- $W_{app} = m(g-a)$. Here $a = g \Rightarrow W_{app} = 0$
- A pendulum will not vibrate in an artificial satellite since $g = 0$ inside the satellite. Therefore

  $$T = 2\pi\sqrt{\frac{l}{g}} = \infty$$

  $\Rightarrow$ Frequency=0

Condition of Weightlessness in a Satellite:

- The force acting on the astronaut of mass ‘m’ is $\frac{GMm}{r^2} - F_R = \frac{mv_0^2}{r}$ here F_R is the reactional force
- The reactional force on the floor of the satellite is zero, hence there is the state of weight lessness in a satellite. i.e., $\frac{GMm}{r^2} = \frac{mv_0^2}{r}$
- The effective acceleration due to gravity inside the satellite = g – a

  $\Rightarrow$ The effective weight = m(g – a)

- As the frame of reference attached to the satellite is an accelerated frame, whose acceleration towards the centre of the earth is $a=\frac{v_0^2}{r}=\frac{GM}{r^2}=g$
- The effective weight of a body in the satellite is zero and independent of the radius of the orbit. Weight lessness is experienced only when the effective gravitational attraction on the astronaut is negligible. The effective gravitational attraction is negligible if the satellite mass itself is less. This is the reason why an astronaut does not feel weight less ness on moon despite moon being a satellite of earth, whereas he feels weightless in an artificial satellite.
- The concept of up and down vanishes in the state of weight lessness. Hence the astronauts are provided with food in the form of paste, filled in tubes.
- If a body is suspended by a string in a satellite, then the tension in the string is zero.
- In this condition, all bodies lying inside the satellite are floating in spaces. Consequently an astronaut cannot drink water with the help of a glass.
- Self potential energy of a thin uniform shell of mass 'M' and radius 'R"
- For a sphere of radius 'x', mass of the sphere

$=\frac{4}{3}\pi x^3\rho$ where ρ = density of sphere

Gravitational potential of the surface

$=-\frac{4}{3}\pi G\rho x^2$

(since gravitational potential

$=-\frac{Gm}{x}$

$=\frac{-G}{x}\times\frac{4}{3}\pi x^3\rho=\frac{-4}{3}\pi Gx^2\rho$)

Work done by the agent in increasing the surface from x to $x+dx$ is

$\frac{-Gm(dm)}{x}$ =Gravitational potential $\times$ dm

$$=\left(\frac{-4}{3}\pi Gx^2\rho\right)\left(4\pi x^2dx\rho\right)=\frac{16\pi^2}{3}G\rho^2x^4dx$$

Therefore total work done

$$\Rightarrow\ =\frac{-16\pi^2}{3}G\rho^2\int_0^R x^4dx=\frac{-16\pi^2G\rho^2R^5}{15}$$

$$=\frac{-16\pi^2GR^5}{15}\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2=\frac{-3}{5}\frac{GM^2}{R}$$

= Gravitational self potential energy of a shell

CONCEPTUAL QUESTIONS

NEWTON'S LAW OF GRAVITATION ANDGRAVITATIONAL INTENSITIES

1. If F_g and F_e are gravitational and electrostatic forces between two electrons at a distance 0.1 m then F_g / F_e is in the order of

1) 10^{43} 2) 10^{-43} 3) 10^{35} 4) 10^{-35}

2. $F=\frac{Gm_1m_2}{r^2}$ **is valid**

1) Between bodies with any shape
2) Between particles
3) Between any bodies with uniform density
4) Between any bodies with same shape

3. F_g, F_e and F_n represent the gravitational, electro-magnetic and nuclear forces respectively, then arrange the increasing order of their strengths

1) F_n, F_e, F_g 2) F_g, F_e, F_n
3) F_e, F_g, F_n 4) F_g, F_n, F_e

4. Find the false statement

1) Gravitational force acts along the line joining the two interacting particles
2) Gravitational force is independent of medium
3) Gravitational force forms an action-reaction pair
4) Gravitational force does not obey the principle of superposition.

5. **Law of gravitation is not applicable if**
A) Velocity of moving objects are comparable to velocity of light
B) Gravitational field between objects whose masses are greater than the mass of sun.
1) A is true, B is false
2) A is false, B is true
3) Both A & B are true
4) Both A&B are false

6. **Among the following find the wrong statement**
1) Law of gravitation is framed using Newton's third law of motion
2) Law of gravitation cannot explain why gravity exists
3) Law of gravitation does not explain the presense of force even when the particles are not in physical contact
4) When the range is long, gravitational force becomes repulsive.

7. **Out of the following interactions the weakest is**
1) gravitational
2) electromagnetic
3) nuclear
4) electrostatic

8. **Neutron changing into Proton by emiting electron and anti neutrino this due to**
1) Gravitiaonal Forces
2) Electro magnetic Forces
3) Weak Nuclear Forces
4) Strong Nuclear Forces

9. **Attractive Force is exists between two protons inside the Nucleous this is due to**
1) Gravitiaonal Forces
2) Electro magnetic Forces
3) Weak Nuclear Forces
4) Strong Nuclear Forces

10. **Repulsive force exist between two protons out side the nucleous this due to**
1) Gravitiaonal Forces
2) Electro magnetic Forces
3) Weak Nuclear Forces
4) Strong Nuclear Forces

11. **Radio activity decay exist due to**
1) Gravitiaonal Forces
2) Electro magnetic Forces
3) Weak Nuclear Forces
4) Strong Nuclear Forces

12. **Two equal masses separated by a distance d attract each other with a force (F). If one unit of mass is transferred from one of them to the other, the force**
1) does not change
2) decreases by (G/d^2)
3) becomes d^2 times
4) increases by $(2G/d^2)$

13. **Which of the following is the evidence to show that there must be a force acting on earth and directed towards Sun?**
1) Apparent motion of sun around the earth
2) Phenomenon of day and night
3) Revolution of earth round the Sun
4) Deviation of the falling body towards earth

14. **Intensity of gravitational field inside the hollow spherical shell is**
1) Variable 2) minimum
3) maximum 4) zero

15. **Six particles each of mass 'm' are placed at the corners of a regular hexagon of edge length 'a'. If a point mass 'm_0' is placed at the centre of the hexagon, then the net gravitational force on the point mass 'm_0' is**
1) $\frac{6Gm^2}{a^2}$ 2) $\frac{6Gmm_0}{a^2}$
3) zero 4) none of these

16. **If suddenly the gravitational force of attraction between earth and satellite revolving around it becomes zero, then the satellite will (AIEEE 2002)**
1) Continue to move in its orbit with same velocity
2) Move tangential to the original orbit with the same velocity
3) Becomes sationary in its orbit
4) Move towards the earth

KEPLER'S LAWS

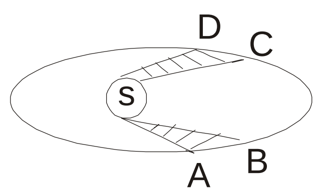
17. The time period of an earth's satellite in circular orbit is independent of
1) the mass of the satellite
2) radius of its orbit
3) both the mass and radius of the orbit
4) neither the mass of the satellite nor the radius of its orbit

18. If the earth is at one-fourth of its present distance from the sun, the duration of the year would be
1) Half the present year
2) One-eigth the present year
3) One-fourth the present year
4) One -sixteenth the present year

19. The radius vector drawn from the sun to a planet sweeps out ___ areas in equal time
1) equal 2) unequal
3) greater 4) less

20. If the area swept by the line joining the sun and the earth from Feb 1 to Feb 7 is 'A', then the area swept by the radius vector from Feb 8 to Feb 28 is
1) A 2) 2A 3) 3A 4) 4A

21. The motion of a planet around sun in an elliptical orbit is shown in the following figure. Sun is situated on one focus. The shaded areas are equal. If the planet takes time 't_1' and 't_2' in moving from A to B and from C to D respectively, then



1) $t_1 > t_2$
2) $t_1 < t_2$
3) $t_1 = t_2$
4) incomplete information

RELATION BETWEEN G AND G, VARIATION OF G WITH ALTITUDE, DEPTH AND ROTATION OF EARTH

22. If the speed of rotation of earth about its axis increases, then the weight of the body at the equator will
1) increase
2) decrease
3) remainunchanged
4) sometimesdecrease and sometimes increase

23. The ratio of acceleration due to gravity at a depth 'h' below the surface of earth and at a height 'h' above the surface for h<<R
1) constant only when h<<R
2) increases linearly with h
3) increases parabolically with h
4) decreases

24. If the gravitational force of earth suddenly disappears, then which of the following is correct?
1) weight of the body is zero
2) mass of the body is zero
3) both mass and weight become zero
4) neither the weight nor the mass is zero

25. Which of the following quantities remain constant in a planetory motion, when seen from the surface of the sun.
1) K.E
2) angular speed
3) speed
4) Angular momentum

26. Average density of the earth
1) does not depend on 'g'
2) is a complex function of 'g'
3) is directly proportional to 'g'
4) is inversely proportional to 'g'

27. A person will get more quantity of matter in Kg-Wt at
1) poles 2) at lattitude of 60^0
3) equator 4) satellite

28. A pendulum clock which keeps correct time at the surface of the earth is taken into a mine, then
1) it keeps correct time 2) it gains time
3) it loses time 4) none of these

29. Two identical trains A and B move with equal speeds on parallel tracks along the equator. A moves from east to west and B moves from west to east. Which train will exert greater force on the track?

1) A 2) B

3) they will exert equal force

4) The mass and the speed of each train must be known to reach a conclusion.

30. Assuming the earth to be a sphere of uniform density the acceleration due to gravity

1) at a point outside the earth is inversely proportional to the square of its distance from the centre

2) at a point outside the earth is inversely proportional to its distance from the centre

3) at a point inside is zero

4) at a point inside is inversely proportional to its distance from the centre.

31. If earth were to rotate faster than its present speed, the weight of an object

1) increase at the equator but remain unchanged at poles

2) decrease at the equator but remain unchanged at the poles

3) remain unchanged at the equator but decrease at the poles

4) remain unchanged at the equator but increase at the poles

32 The time period of a simple pendulum at the centre of the earth is

1) zero 2) infinite

3) less than zero 4) none of these

33. A body of mass 5 kg is taken into space. Its mass becomes.

1) 5 kg 2) 10 kg

3) 2 kg 4) 30 kg

34. If the mean radius of earth is R, its angular velocity is ω and the acceleration due to gravity at the surface of the earth is 'g' then the cube of the radius of the orbit of a satellite will be

1) $\frac{Rg}{\omega^2}$ 2) $\frac{R^2g}{\omega}$ 3) $\frac{R^2g}{\omega^2}$ 4) $\frac{R^2\omega}{g}$

35. If R=radius of the earth and g =acceleration due to gravity on the surface of the earth, the acceleration due to gravity at a distance (r<R) from the centre of the earth is proportional to

1) r 2) r^2

3) r^{-2} 4) r^{-1}

36. If R = radiusof the earth and g = acceleration due to gravity on the surface of the earth, the acceleration due to gravity at a distance (r>R) from the centre of the earth is proportional to

1) r 2) r^2

3) r^{-2} 4) r^{-1}

37. Earth is flattend at poles and bulging at equators this is due to

1) revolution of earth around the sun is an elliptical orbit

2) angular of velocity of spining about its axis is more at equator

3) centrifugal force is more at equator than poles

4) more centrifugal force at poles than equator

38. The tidal waves in the sea are primarily due to

1) the gravitational effect of the moon on the earth

2) the gravitational effect of the sun on the earth

3) the gravitational effect of the venus on the earth

4) the atmospheric effect of the earth itself

39. Consider earth to be a homogeneous sphere. Scientist A goes deep down in a mine and scientist B goes high up in a baloon. The gravitational field measured by

1) A goes on decreasing and that of B goes on increasing

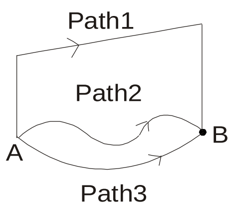
2) B goes on decreasing and that of A goes on increasing

3) Each decreases at the same rate

4) Each decreases at different rates.

SATELLITE MOTION, GRAVITATIONAL POTENTIAL, GRAVITATIONAL POTENTIAL ENERGY AND WORK DONE

40. A gravitation field is present in a region. A point mass is shifted from A to B, along different paths shown in the figure. If W_1, W_2 and W_3 represent the work done by gravitational force for respective paths, then



1) $W_1 = W_2 = W_3$ 2) $W_1 > W_2 > W_3$
3) $W_1 > W_3 > W_2$ 4) none of these

41. The minimum number of geostationary satellites required to televise a programme all over the earth is
1) 2 2) 6 3) 4 4) 3

42. When a satellite going around the earth in a circular orbit of radius r and speed v loses some of its energy , then
1) r and v both increase
2) r and v both decrease
3) r will increase and v will decrease
4) r will decrease and v will increase

43. The satellite is orbiting a planet at a certain height in a circular orbit. If the mass of the planet is reduced to half, the satellite would
1) fall on the planet
2) go to orbit of smaller radius
3) go to orbit of higher radius
4) escape from the planet

44. A satellite is revolving round the earth in an elliptical orbit. It speed will be
1) same at all points of the orbit
2) different at different point of the orbit
3) maximum at the farthest point
4) minimum at the nearest point

45. An artificial satellite of the earth releases a packet. If air resistance is neglected, the point where the packet will hit, will be
1) a head
2) exactly below
3) behind
4) it will never reach the earth

46. A satellite is moving in a circular orbit round the earth. If any other planet comes in between then it
1) Continuous to move with the same speed along the same path
2) Moves with the same velocity tangential to original orbit.
3) Falls down with increasing velocity.
4) Comes to rest after moving certain distance along original path.

47. A space-ship entering the earth's atmosphere is likely to catch fire. This is due to
1) The surface tension of air
2) The viscosity of air
3) The high temperature of upper atmosphere
4) The greater portion of oxygen in the atmosphere at greater height.

48. An astronaut orbiting the earth in a circular orbit 120 km above the surface of earth, gently drops a ball from the space-ship. The ball will
1) Move randomly in space
2) Move along with the space-ship
3) Fall vertically down to earth
4) Move away from the earth

49. Following physical quantity of a planet that revolves around Sun in an elliptical orbit is constant.
1) Kinetic energy 2) Potential energy
3) Angular momentum 4) Linear velocity

50. A geostationary satellite has an orbital period of
1) 2 hours 2) 6 hours
3) 24 hours 4) 12 hours

51. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then

1) its angular momentum about the earth decreases
2) its kinetic energy decreases
3) its kinetic energy remains constant
4) its period of revolution around the earth increases

52. The period of a satellite moving in circular orbit near the surface of a planet is independent of

1) mass of the planet 2) radius of the planet
3) mass of the satellite 4) density of planet

53. Out of the following statements, the one which correctly describes a satellite orbiting about the earth is

1) There is no force acting on the satellite
2) The acceleration and velocity of the satellite are roughly in the same direction
3) The satellite is always accelerating about the earth
4) The satellite must fall, back to earth when its fuel is exhausted.

54. When an astronaut goes out of his space-ship into the space he will

1) Fall freely on the earth
2) Go upwards
3) Continue to move along with the satellite in the same orbit.
4) Go spiral to the earth

55. When the height of a satellite increases from the surface of the earth.

1) PE decreases, KE increases
2) PE decreases, KE decreases
3) PE increases, KE decreases
4) PE increases, KE increases

56. The energy required to remove an earth satellite of mass 'm' from its orbit of radius 'r' to infinity is

1) $\frac{GMm}{r}$ 2) $\frac{-GMm}{2r}$
3) $\frac{GMm}{2r}$ 4) $\frac{Mm}{2r}$

57. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth

1) the acceleration of S is always directed towards the centre of the earth
2) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
3) the total mechanical energy of S varies periodically with time
4) the linear momentum of S remains constant in magnitude

58. The time period of revolution of geostationary satellite with respect to earth is

1) 24 hrs 2) 1 year
3) Infinity 4) Zero

59. A relay satellite transmits the television programme from one part of the world to another part continuously because its period

1) is greater than the period of the earth about its axis
2) is less than period of rotation of the earth about its axis.
3) has no relation with the period of rotation of the earth about its axis.
4) is equal to the period of rotation of the earth about its axis.

60. A synchronous satellite should be at a proper height moving

1) From West to East in equatorial plane
2) From South to North in polar plane
3) From East to West in equatorial plane
4) From North to South in polar plane

61. The orbital angular velocity vector of a geostationary satellite and the spin angular velocity vector of the earth are

1) always in the same direction
2) always in opposite direction
3) always mutually perpendicular
4) inclined at 23 1/2° to each other

62. A satellite is revolving round the earth. Its kinetic energy is E_k. How much energy is required by the satellite such that it escapes out of the gravitation field of earth

1) $2E_k$ 2) $3E_k$ 3) $\frac{E_k}{2}$ 4) infinity

63. The work done by an external agent to shift a point mass from infinity to the centre of the earth is 'W'. Then choose the correct relation.

1) W=0 2) W>0 3) W<0 4) $W \leq 0$

64. Where is the intensity of the gravitational field of the earth maximum?

1) centre of earth 2) equator
3) poles 4) same everywhere

65 Let V_G and E_G denote gravitational potential and field respectively, then choose the wrong statement.

1) $V_G = 0, E_G = 0$ 2) $V_G \neq 0, E_G = 0$
3) $V_G = 0, E_G \neq 0$ 4) $V_G \neq 0, E_G \neq 0$

66. For a satellite moving in an orbit around the earth, the ratio of K.E to P.E is

1) $-\frac{1}{2}$ 2) $-\frac{1}{\sqrt{2}}$ 3) -2 4) $-\sqrt{2}$

67. Two identical spherical masses are kept at some distance. Potential energy when a mass 'm' is taken from the surface of one sphere to the other

1) increases continuously
2) decreases continuously
3) first increases, then decreases
4) first decreases, then increases

68. A thin spherical shell of mass 'M' and radius 'R' has a small hole. A particle of mass 'm' is released at the mouth of them. Then

1) the particle will execute S.H.M inside the shell
2) the particle will oscillate inside the shell, but the oscillations are not simple harmonic
3) the particle will not oscillate, but the speed of the particle will go on increasing
4) none of these

69. For a satellite projected from the earth's surface with a velocity greater than orbital velocity the nature of the path it takes when its energy is negative, zero and positive respectively is

1) Elliptical, parabolic and hyperbolic
2) Hyperbolic, parabolic and elliptical
3) Elliptical, circular and parabolic
4) Parabolic, circular and Elliptical

70. If a satellite is moved from one stable circular orbit to a farther stable circular orbit, then the following quantity increases

1) Gravitational force
2) Gravitational P.E.
3) linear orbital speed
4) Centripetal acceleration

71. The gravitational field is a conservative field. The work done in this field by moving an object from one point to another

1) depends on the end-points only
2) depends on the path along which the object is moved
3) depends on the end-points as well as the path between the points.
4) is not zero when the object is brought back to its initial position.

72. A hole is drilled through the earth along a diameter and a stone is dropped into it. When the stone is at the centre of the earth, it has finite a) weight b) acceleration c) P.E. d) mass

1) a & b 2) b & c 3) a, b & c 4) c & d

73. If the universal gravitational constant decreases uniformly with time, then a satellite in orbit will still maintain its

1) weight 2) tangential speed
3) period of revolution 4) angular momentum

74. A body has weight (w) on the ground. The work which must be done to lift it to a height equal to the radius of the earth is

1) equal to WR 2) greater than WR
3) less than WR 4) we can't say

ESCAPE SPEED

75. The earth retains its atmosphere. This is due to
1) The special shape of the earth
2) The escape velocity being greater than the mean speed of the molecules of the atmospheric gases.
3) The escape velocity being smaller than the mean speed of the molecules of the atmospheric gases.
4) The sun's gravitational effect.

76. Ratio of the radius of a planet A to that of planet B is 'r'. The ratio of accelerations due to gravity for the two planets is x. The ratio of the escape velocities from the two planets is
1) $\sqrt{rx}$ 2) $\sqrt{r/x}$
3) $\sqrt{r}$ 4) $\sqrt{x/r}$

77. The ratio of the escape velocity and the orbital velocity is
1) $\sqrt{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) 2 4) 1/2

78. The escape velocity from the earth for a rocket is 11.2 km/sec. Ignoring the air resistance, the escape velocity of 10 mg grain of sand from the earth will be
1) 0.112 km/sec 2) 11.2 km/sec
3) 1.12 km/sec 4) None

79. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45^0 with the vertical, the escape velocity will be
1) $11\sqrt{2}\,\text{km/s}$ 2) $22\,\text{km/s}$
3) $11\,\text{km/s}$ 4) $11\sqrt{2}\,\text{km/s}$

80. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energies is
1) Positive 2) Negative
3) Zero
4) May be positive or negative depending upon its initial velocity

81. The escape velocity of a body depends upon its mass as
1) m^0 2) m^1 3) m^3 4) m^2

82. The magnitude of potential energy per unit mass of the object at the surface of earth is 'E'. Then escape velocity of the object is
1) $\sqrt{2E}$ 2) $4E^2$
3) $\sqrt{E}$ 4) $\sqrt{E/2}$

83. A space station is set up in space at a distance equal to earth's radius from earth's surface. Suppose a satellite can be launched from space station. Let V_1 and V_2 be the escape velocites of the satllite on earth's surface and space station respectively. Then
1) $V_2 = V_1$ 2) $V_2 < V_1$
3) $V_2 > V_1$ 4) No relation

EARTH SATELLITES

84. If the universal gravitational constant increases uniformly with time, then a satellite in orbit will still maintain its
1) weight 2) tangential speed
3) period of revolution
4) conservation angular momentum

85. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving around earth in circular orbits of radii r_1 and r_2 ($r_1 > r_2$) respectively. Which of the following statements is true regarding their velocities V_1 and V_2.
1) $V_1 = V_2$ 2) $V_1 < V_2$
3) $V_1 > V_2$ 4) $\frac{V_1}{r_1} = \frac{V_2}{r_2}$

86. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change
1) gravitational potential energy
2) angular vleocity
3) linear orbital velocity
4) centripetal acceleration

87. An artificial satellite of the earth releases a packet. If air resistance is neglected, the point where the packet will hit, will be

1) a head
2) exactly below
3) behind
4) it will never reach the earth

88. A planet of mass 'm' is in a elliptical orbit about the sun $(m << M)$ with an orbital time period 'T'. If 'A' be the area of the orbit then its angular momentum is

1) $\frac{2mA}{T}$ 2) mAT
3) $\frac{mA}{2T}$ 4) $2mAT$

89. A planet is revolving around the sun in an elliptical orbit the work done on the planet by the gravatational force of the sun is zero
a) in some part of orbit
b) in no part of orbit
c) in one complete revolution
d) in any part of orbit

1) a,b 2) b,c 3) c,d 4) a,c

90. If satellite is orbitting in space having air and no energy being supplied, then path of that satellite would be

1) circular
2) elliptical
3) spiral of increasing radius
4) spiral of decreasing radius

91. A satellite in vacuum

1) is kept in orbit by solar energy
2) prevous energy from gravitational field
3) by remote control
4) No energy is required for revolving

92. Two heavenly bodies s_1 & s_2 not far off from each other, revolve in orbit

1) arround their common centre of mass
2) s_1 is fixed and s_2 revolves around s_1
3) s_2 is fixed and s_1 revolves around s_2
4) cannot say

93. If V, T, L, K and r denote speed, time period, angular momentum, kinetic energy and radius of satellite in circular obrit

a) $V\alpha r^{-1}$ **b)** $L\alpha r^{1/2}$
c) $T\alpha r^{3/2}$ **d)** $K\alpha r^{-2}$

1) a,b are true 2) b,c are true
3) a,b,d are true 4) a,b,c are true

94. Two similar satellite s_1 and s_2 of same mass 'm' have completely inelastic collision while orbitting earth in the same circular orbit in opposite direction then

1) total energy of satellites and earth system become zero
2) the satellites stick together and fly into space
3) the combined mass falls vertically down
4) the satellites move in opposite direction

ENEGRY OF ORBITTING SATELLITES

95. For a planet revolving round the sun, when it is nearest to the sun is

1) K.E. is min and P.E. is max.
2) Both K.E. and P.E. are min
3) K.E. is max. and P.E. is min
4) K.E. and P.E. are equal

96. A body is dropped from a height equal to radius of the earth. The velocity acquired by it before touching the ground is

1) V=$\sqrt{2gR}$ 2) V=3gR
3) V=$\sqrt{gR}$ 4) V=2gR

97. When projectle attains escape velocity, then on the surface of planet , its

1) $KE > PE$ 2) $PE > KE$
3) $KE = PE$ 4) $KE = 2PE$

98. A satellite is moving with constant speed 'V' is a circular orbit about earth. The kinetic energy of the satellite is

1) $\frac{1}{2}mV^2$ 2) mV^2
3) $\frac{3}{2}mV^2$ 4) $2mV^2$

GEOSTATIONARY AND POLAR SATELLITES

99. **The orbit of geo-stationary satellite is circular, the time period of satellite depends on (2008 E)**
1) mass of the Earth
2) radius of the orbit
3) height of the satellite from the surface of Earth
4) all the above

WEIGHTLESSNESS

100. **Pseudo force also called fictitious force such as centrifugal force arises only in**
1) Inertial frames
2) Non-intertial frames
3) Both inertial and non-inertial frames
4) Rigid frames

101. **Feeling of weightlessness in a satellite is due to**
1) absence of inertia
2) absence of gravity
3) absence of accelarating force
4) free fall of satellite

MISCELLANEOUS

102. **The unit of the quantity g/G in SI will be**
1) $Kg\,m^{-2}$ 2) $m\,Kg^{-2}$
3) $m^2\,Kg^{-1}$ 4) $Kg^2 m^{-1}$

103. **Two artificial satellites are revolving in the same circular orbit. Then they must have the same**
1) Mass
2) Angular momentum
3) Kinetic energy
4) Period of revolution

104. **In some region, the gravitational field is zero. Then gravitational potential at that region**
1) Must be zero 2) Can not be zero
3) Must be constant 4) None

105. **A satellite launching station should be**
1) Near the equatorial region
2) Near the polar region
3) On the polar axis
4) At any place

106. **If S_1 is surface satellite and S_2 is geostationary satellite, with time periods T_1 and T_2, orbital velocities V_1 and V_2,**
1) $T_1 > T_2; V_1 > V_2$
2) $T_1 > T_2; V_1 < V_2$
3) $T_1 < T_2; V_1 < V_2$
4) $T_1 < T_2; V_1 > V_2$

107. **If there were a smaller gravitational effect which of the following forces do you think would alter in some respect?**
1) Viscous forces
2) Archimedes uplift
3) Electrostatic
4) Magnetic

108. **Which of the following statements is correct about the motion of earth satellite?**
1) It is always accelerating towards the earth
2) There is no force acting on the satellite
3) Move away from the earth normally to the orbit
4) Fall down on to the earth

109. **According to kepler's second law, line joining the planet to the sun sweeps out equal areas in equal time intervals. This suggests that for the planet.**
1) Radial acceleration is zero
2) Tangential acceleration is zero
3) Transverse acceleration is zero
4) All

110. **Assume that earth is a spherical planet of uniform density ρ, radius R_e, mass M_e and acceleration due to gravity g .Then the gravitaional constant G can be written as**
1) $\frac{3g}{4\pi\rho R_e}$ 2) $\frac{gR_e^2}{M_e}$
3) $\frac{3g}{4\pi\,\rho\,R_e^2}$ 4) $\frac{12\rho\,g}{4\pi\,R_e}$

111. The acceleration due gravity on moon is only one sixth that on earth. Ratio of moon radius $\left(R_m\right)$ to earth's radius $\left(R_e\right)$ should be

1) $\frac{6}{1}$ if both are assumed to have same density

2) $\frac{1}{6}$ if both are assumed to have same density

3) $\frac{5}{18}$ if $\frac{\rho_e}{\rho_m}$ is given as $\frac{5}{3}$.

4) $\frac{9}{5}$ if $\frac{\rho_m}{\rho_e}$ is given as $\frac{3}{5}$

112. Two satellites are revolving around the earth in circular orbits of same radii. Mass of one satellite is 100 times that of the other. Then their periods of revolution are in the ratio

1) 100:1 2) 1:100
3) 1:1 4) 10:1

113. A tunnel is dug along a diameter of the earth. The force on a particle of mass m placed in the tunnel at a distance R/3 from the centre is F. Here R is radius of the earth. Then the force on the same particle when it is on the surface of the earth is (assume the earth as a perfect sphere)

1) 2F/3 2) F/3
3) 3F 4) 3F/2

114. An artificial satellite of mass m is revolving round the earth in a circle of radius R. Then work done in one revolution is

1) mgR 2) $\frac{mgR}{2}$
3) $2\pi R\times mg$ 4) Zero

115. As a body is taken from the surface of earth to another planet.

1) Its weight gradually increases
2) Its weight gradually decreases
3) Its weight gradually increases reaches a maximum value and then decreases
4) Its weight gradually, decreases becomes zero and then increases

116. If T is the period of revolution of the moon round the earth and radius of the orbit is R, then the acceleration of the moon towards the earth is

1) $\frac{4\pi^2 R}{T^2}$ 2) $\frac{T^2}{4\pi^2 R}$
3) $\frac{R}{4\pi^2 T^2}$ 4) $\frac{4\pi^2}{T^2 R^2}$

117. It is not possible to keep a geostationary satellite over Delhi, since Delhi

1) Is not present in A.P
2) Is capital of India
3) Is not in the equatorial plane of the earth
4) All

118. If mass of the earth were quadrupled, to keep the acceleration due to gravity constant on the earth, its radius

1) Must be doubled
2) Should remain constant
3) Must be halved
4) Must be be quadrupled

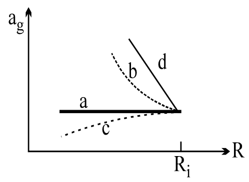
119. When a satellite falls into an orbit of smaller radius its speed

1) decreases 2) increases
3) does not change 4) zero

120. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

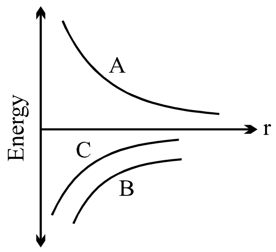
1) increases 2) decreases
3) remains same
4) during the compression increases then returns at the previous value.

121. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse?



1) a 2) b 3) c 4) d

122. The figure shows the variation of energy with the orbit radius of a body in circular planetary motion. Find the correct statement about the curves A, B and C



1) A shows the kinetic energy, B the total energy and C the potential energy of the system.
2) C shows the total energy, B the kinetic energy and A the potential energy of the system.
3) C and A are kinetic and potential energies respectively and B is the total energy of the system.
4) A and B are kinetic and potential energies and C is the total energy of the system.

123. A satellite of mass 5M orbits the earth in a circular orbit. At one point in its orbit, the satellite explodes into two pieces, one of mass M and the other of mass 4M. After the explosion the mass M ends up travelling in the same circular orbit, but in opposite direction. After explosion the mass 4M is
1) In a circular orbit
2) unbound
3) elliptical orbit
4) data is insufficient to determine the nature of the orbit.

KEY

CONCEPTUAL

1) 2	2) 2	3) 2	4) 4	5) 3
6) 4	7) 1	8) 3	9) 4	10) 2
11) 3	12) 2	13) 3	14) 4	15) 3
16) 2	17) 1	18) 2	19) 1	20) 3
21) 3	22) 2	23) 2	24) 1	25) 4
26)3	27) 3	28) 3	29) 1	30) 1
31) 2	32) 2	33) 1	34) 3	35) 1
36) 3	37) 3	38) 1	39) 4	40) 1
41) 4	42) 4	43) 4	44) 2	45) 4
46) 2	47) 2	48) 2	49) 3	50) 3
51) 1	52) 3	53) 3	54) 3	55) 3
56) 3	57) 1	58) 3	59) 4	60) 1
61) 1	62) 1	63) 3	64) 3	65)3
66) 1	67) 3	68) 4	69) 1	70) 2
71) 1	72) 4	73) 4	74) 3	75) 2
76) 1	77) 1	78) 2	79) 3	80) 2
81) 1	82) 1	83) 2	84) 4	85) 2
86) 1	87) 4	88) 1	89) 4	90) 4
91) 4	92) 1	93) 2	94) 3	95) 3
96) 3	97) 3	98) 1	99) 4	100) 2
101) 4	102) 1	103) 4	104) 3	105) 1
106) 4	107) 2	108) 1	109) 3	110) 1
111) 2	112) 3	113) 3	114) 4	115) 4
116) 1	117) 3	118) 1	119) 2	120) 2
121) 2	122) 4	123) 2		

LEVEL-I

KEPLER'S LAWS

1. If 'A' is areal velocity of a planet of mass M, its angular momentum is
1) M/A
2) 2MA
3) A^2M
4) AM^2

2. Let 'A' be the area swept by the line joining the earth and the sun during Feb 2012. The area swept by the same line during the first week of that month is
1) A/4
2) 7A/29
3) A
4) 7A/30

3. The distance of Neptune and saturn from the Sun are respectively. 10^{13} and 10^{12} meters and their periodic times are respectively T_n and T_s. If their orbits are assumed to be circular, the value of T_n / T_s is
1) 100
2) $10\sqrt{10}$
3) $\frac{1}{10\sqrt{10}}$
4) 10

4. A planet moves around the sun in elliptical orbit . When earth is closest from the sun, it is at a distance r having a speed V. When it is at a distance 4r from the sun, its speed is
1) 4v
2) $\frac{v}{4}$
3) 2v
4) $\frac{v}{2}$

5. **If G is the universal gravitaional constant and ρ is the uniform density of a spherical planet. Then shortest possible period of rotation of the planet can be**

1) $\sqrt{\frac{\pi G}{2\rho}}$ 2) $\sqrt{\frac{3\pi G}{\rho}}$

3) $\sqrt{\frac{\pi}{6G\rho}}$ 4) $\sqrt{\frac{3\pi}{G\rho}}$

6. **The ratio of Earth's orbital angular momentum (about the sun) to its mass is 4 .4 X 10^{15} m^2s^{-1}. The area enclosed by the earth's orbit is approximately**

1) 1 X 10^{23} m^2 2) 3 X 10^{23} m^2

3) 5 X 10^{23} m^2 4) 7 X 10^{23} m^2

7. **Suppose the gravitational force varies inversely as the n^{th} power of distance, then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to [AIEEE -2004]**

1) $R^{\left(\frac{n+1}{2}\right)}$ 2) $R^{\left(\frac{n-2}{2}\right)}$

3) R^n 4) $R^{\left(\frac{n-1}{2}\right)}$

LAW OF GRAVITATION

8. **Two metal spheres each of radius 'r' are kept in contact with each other. If d is the density of the material of the sphere, then the gravitational force between those spheres is proportional to**

1) d^2r^6 2) d^2r^4

3) $\frac{d^2}{r^4}$ 4) $\frac{r^2}{d^2}$

9. **The gravitational force between two identical objects at a separation of 1m is 0.0667mg wt. The masses of the objects**

($G = 6.67\times10^{-11} Nm^2/kg^2$ **and** $g = 10 m/s^2$**)**

1) 200kg, 200kg
2) 100kg, 100 kg
3) 300kg, 300kg
4) 400kg, 400kg

10. **Three particles of identical masses 'm' are kept at the vertices of an equilateral triangle of each side length 'a'. The gravitational force of attraction on any one of the particles is**

1) $\sqrt{2}\frac{Gm^2}{a^2}$ 2) $\sqrt{3}\frac{Gm^2}{a^2}$

3) $\frac{3Gm^2}{a^2}$ 4) $\frac{2Gm^2}{a^2}$

11. **If the mass of one particle is increased by 50 % and the mass of another particle is decreased by 50 %, the force between them**

1) decreases by 25% 2) decreases by 75 %
3) increases by 25% 4) does not change

12. **Mass M is divided into two parts X and (1–X). For a given seperation the value of X for which the gravitational attraction between the two pieces becomes maximum is**

1) 1/2 2) 3/5 3) 1 4) 2

13. **Two particles of equal mass go around in a circle of radius 'r'under the action of their mutual gravitational attraction. If the mass of each particle is m, the speed of each particle is**

1) $\sqrt{\frac{Gm}{r}}$ 2) $\sqrt{\frac{Gm}{2r}}$ 3) $\sqrt{\frac{Gm}{4r}}$ 4) $\sqrt{\frac{2Gm}{r}}$

14. **A 3 kg mass and a 4 kg mass are placed on x and y axes at a distance of 1 metre from the origin and a 1 kg mass is placed at the origin. Then the resultant gravitational force on 1 kg mass is**

1) 7G 2) G 3) 5G 4) 3G

ACCELERATION DUE TO GRAVITY AND ITS VARIATION WITH ALTITUDE, DEPTH AND ROTATION OF EARTH

15. **If g on the surface of the earth is 9.8** m/s^2**, its value at a height of 6400 km is (Radius of the earth = 6400km).**

1) $4.9ms^{-2}$ 2) $9.8ms^{-2}$
3) $2.45ms^{-2}$ 4) $19.6ms^{-2}$

16. If g on the surface of the earth is $9.8 ms^{-2}$, its value at a depth of 3200km (Radius of the earth = 6400km) is

1) $9.8 ms^{-2}$ 2) zero
3) $4.9 ms^{-2}$ 4) $2.45 ms^{-2}$

17. If mass of the planet is 10% less than that of earth and radius of the planet is 20% greater than that of earth then the weight of 40kg person on that planet is

1) 10 kg wt 2) 25 kg wt
3) 40 kg wt 4) 60 kg wt

18. The angular velocity of the earth with which it has to rotate so that the acceleration due to gravity on 60^0 latitude becomes zero is

1) $2.5\times10^{-3}\ rad\ s^{-1}$ 2) $1.5\times10^{-3}\ rad\ s^{-1}$
3) $4.5\times10^{-3}\ rad\ s^{-1}$ 4) $0.5\times10^{-3}\ rad\ s^{-1}$

19. Assume that the acceleration due to gravity on the surface of the moon is 0.2 times the acceleration due to gravity on the surface of the earth. If R_e is the maximum range of a projectile on the earth's surface, what is the maximum range on the surface of the moon for the same velocity of projection

1) $0.2R_e$ 2) $2R_e$
3) $0.5R_e$ 4) $5R_e$

20. A particle hanging from a spring stretches it by 1 cm at earth's surface. Radius of earth is 6400 km. At a place 800 km above the earth's surface, the same particle will stretch the spring by

1) 0.79 cm 2) 1.2 cm
3) 4 cm 4) 17 cm

21. A tunnel is dug along a diameter of the earth. The force on a particle of mass 'm' placed in the tunnel at a distance x from the centre is

1) $\frac{GM_e m}{R^3}x$ 2) $\frac{GM_e m}{R^2}x$
3) $\frac{GM_e m}{R^3 x}$ 4) $\frac{GM_e mR^3}{x}$

22. The acceleration due to gravity at the poles is $10 ms^{-2}$ and equatorial radius is 6400km for the earth. Then the angular velocity of rotation of the earth about its axis so that the weight of a body at the equator reduces to 75% is

1) $\frac{1}{1600}$ rads^{-1} 2) $\frac{1}{800}$ rads^{-1}
3) $\frac{1}{400}$ rads^{-1} 4) $\frac{1}{200}$ rads^{-1}

23. The radius and density of two artificial satellites are R_1, R_2 and respectively. The ratio of acceleration due to gravities on them will be

1) $\frac{R_2\rho_2}{R_1\rho_1}$ 2) $\frac{R_1\rho_2}{R_2\rho_1}$
3) $\frac{R_1\rho_1}{R_2\rho_2}$ 4) $\frac{R_2\rho_1}{R_1\rho_2}$

24. What is the percentage change in the value of 'g' on shifting from equator to poles on the earth's surface?

1) 4.5% 2) 0.343% 3) 0.05% 4) 1.5%

GRAVITATIONAL FIELD INTENSITY

25. The point at which the gravitational force acting on any mass is zero due to the earth and the moon system is. (The mass of the earth is approximately 81 times the mass of the moon and the distance between the earth and the moon is 3,85,000km.)

1) 36,000km from the moon
2) 38,500km from the moon
3) 34500km from the moon
4) 30,000 from the moon

26. Masses 2 kg and 8 kg are 18 cm apart. The point where the gravitational field due to them is zero is

1) 6 cm from 8 kg mass
2) 6 cm from 2 kg mass
3) 1.8 cm from 8 kg mass
4) 9 cm from each mass

27. **Particles each of mass M are placed along x-axis at x=1m, x=2m, x=4m, x=8m,..... etc to infinity. Gravitational field strength at the origin due to this system of particles is**

1) 2GM 2) 2GM/3
3) 4GM/3 4) 5GM/4

28. **Particles of masses m_1 and m_2 are at a fixed distance apart. If the gravitational field strength at m_1 and m_2 are $\vec{I}_1$ and $\vec{I}_2$ respectively. Then,**

1) $m_1\overrightarrow{I_1}+m_2\overrightarrow{I_2}=0$ 2) $m_1\overrightarrow{I_2}+m_2\overrightarrow{I_1}=0$
3) $m_1\overrightarrow{I_1}-m_2\overrightarrow{I_2}=0$ 4) $m_1\overrightarrow{I_2}-m_2\overrightarrow{I_1}=0$

GRAVITATIONAL POTENTIAL AND GRAVITATIONAL POTENTIAL ENERGY

29. **The PE of three objects of masses 1kg, 2kg and 3kg placed at the three vertices of an equilateral triangle of side 20cm is**

1) 25G 2) 35G 3) 45G 4) 55G

30. **A small body is initially at a distance 'r' from the centre of earth. 'r' is greater than the radius of the earth. If it takes W joule of work to move the body from this position to another position at a distance 2r measured from the centre of earth, how many joules would be required to move it from this position to a new position at a distance of 3r from the centre of the earth.**

1) W/5 2) W/3 3) W/2 4) W/6

31. **A body of mass 'm' is raised from the surface of the earth to a height 'nR' (R -radius of earth). Magnitude of the change in the gravitational potential energy of the body is (g - acceleration due to gravity on the surface of earth)**

1) $\left(\frac{n}{n+1}\right)mgR$ 2) $\left(\frac{n-1}{n}\right)mgR$

3) $\frac{mgR}{n}$ 4) $\frac{mgR}{(n-1)}$

32. **A person brings a mass 2 kg from A to B. The increase in kinetic energy of mass is 4J and work done by the person on the mass is $-10J$. The potential difference between B and A is .... J/kg**

1) 4 2) 7 3) -3 4) -7

33. **The potential energy of a body of mass 'm' is given by U=px+qy+rz. The magnitude of the acceleration of the body will be**

1) $\frac{p+q+r}{m}$ 2) $\frac{\sqrt{p^2+q^2+r^2}}{m}$

3) $\frac{\sqrt{p^3+q^3+r^3}}{m}$ 4) $\frac{\sqrt{p^4+q^4+r^4}}{m}$

34. **A particle is placed in a field characterized by a value of gravitational potential given by V = -kxy, where 'k' is a constant. If $\overline{E_g}$ is the gravitational field then.**

1) $\overline{E_g}=k\left(x\bar{i}+y\bar{j}\right)$ and is conservative in nature
2) $\overline{E_g}=k\left(y\bar{i}+x\bar{j}\right)$ and is conservative in nature
3) $\overline{E_g}=k\left(x\bar{i}+y\bar{j}\right)$ and is non conservative in nature
4) $\overline{E_g}=k\left(y\bar{i}+x\bar{j}\right)$ and is non conservative in nature

35. **A thin rod of length 'L' is bent to form a semi circle. The mass of the rod is 'M'. What will be the gravitational potential at the centre of the circle?**

1) $\frac{-GM}{L}$ 2) $\frac{-GM}{2\pi L}$

3) $\frac{-\pi GM}{2L}$ 4) $\frac{-\pi GM}{L}$

36. **The work done in shifting a particle of mass 'm' from the centre of earth to the surface of the earth is**

1) –mgR 2) $\frac{1}{2}mgR$ 3) zero 4) mgR

ESCAPE SPEED

37. The ratio of escape velocities of two planets if g value on the two planets are $9.9 m/s^2$ and $3.3 m/s^2$ and their radii are 6400km and 3200km respectively is

1) 2.36 : 1 2) 1.36 : 1
3) 3.36 : 1 4) 4.36 : 1

38. The escape velocity from the surface of the earth of radius R and density ρ

1) $2R\sqrt{\frac{2\pi\rho G}{3}}$ 2) $2\sqrt{\frac{2\pi\rho G}{3}}$

3) $2\pi\sqrt{\frac{R}{g}}$ 4) $\sqrt{\frac{2\pi G\rho}{R^2}}$

39. A body is projected vertically up from surface of the earth with a velocity half of escape velocity. The ratio of its maximum height of ascent and radius of earth is

1) 1 : 1 2) 1 : 2 3) 1 : 3 4) 1 : 4

40. The mass of a planet is half that of the earth and the radius of the planet is one fourth that of earth. If we plan to send an artificial satellite from the planet, the escape velocity will be, $(V_e = 11 kms^{-1})$

1) $11 kms^{-1}$ 2) 5.5 kms^{-1}

3) 15.55 kms^{-1} 4) 7.78 kms^{-1}

41. If a rocket is fired with a velocity, $V = 2\sqrt{gR}$ near the earth's surface and goes upwards, its speed in the inter-stellar space is

1) $4\sqrt{gR}$ 2) $\sqrt{2gR}$ 3) $\sqrt{gR}$ 4) $\sqrt{4gR}$

42. A spaceship is launched into a circular orbit of radius 'R' close to the surface of earth. The additional velocity to be imparted to the spaceship in the orbit to overcome the earth's gravitational pull is (g = acceleration due to gravity)

1) 1.414Rg 2) $1.414\sqrt{Rg}$

3) 0.414Rg 4) $0.414\sqrt{gR}$

43. If d is the distance between the centres of the earth of mass M_1 and moon of mass M_2, then the velocity with which a body should be projected from the mid point of the line joining the earth a

1) $\sqrt{\frac{G(M_1+M_2)}{d}}$ 2) $\sqrt{\frac{G(M_1+M_2)}{2d}}$

3) $\sqrt{\frac{2G(M_1+M_2)}{d}}$ 4) $\sqrt{\frac{4G(M_1+M_2)}{d}}$

44. The escape velocity from the earth is 11 km/sec. The escape velocity from a planet having twice the radius and same density as earth is

1) 22 km/sec 2) 15.5 km/sec
3) 11 km/sec 4) 5.5 km/sec

45. An object of mass 'm' is at rest on earth's surface. Escape speed of this object is V_e. Same object is orbitting earth with $h = R$ then escape speed is V_e^1. Then

1) V_e^1 2) $V_e = 2V_e^1$

3) $V_e = \sqrt{2}V_e^1$ 4) $V_e^1 = \sqrt{2}V_e$

46. A satellite revolves in a circular orbit with speed $V = \frac{1}{\sqrt{3}}V_e$. If satellite is suddenly stopped and allowed to fall freely onto earth, the speed with which it hits earth's surface is

1) $\sqrt{gR}$ 2) $\sqrt{\frac{gR}{3}}$

3) $\sqrt{2gR}$ 4) $\sqrt{\frac{2}{3}gR}$

EARTH SATELLITE

47. The orbital speed for an earth satelite near the surface of the earth is 7 km/sec. If the radius of the orbit is 4 times the radius of the earth, the orbital speed would be

1) 3.5 km/sec 2) 7 km/sec
3) $7\sqrt{2}$ km/sec 4) 14 km/sec

48. Two satellites are revolving round the earth at different heights. The ratio of their orbital speeds is 2 : 1. If one of them is at a height of 100km, the height of the other satellite is

1) 19600km 2) 24600km
3) 29600km 4) 14600km

49. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is [AIEEE-2004]

1) gx 2) $\left(\frac{gR^2}{R+x}\right)^{1/2}$ 3) $\frac{gR^2}{R+x}$ 4) $\frac{gR}{R-x}$

50. The period of revolution of an earth's satellite close to the surface of earth is 60 minutes. The period of another earth's satellite in an orbit at a distance of three times earth's radius from its surface will be

1) 90 minutes 2) $90\times\sqrt{8}$ minutes
3) 270 minutes 4) 480minutes

51. The time period of satellite of earth is 5 hr. If the separation between earth and the satellite is increased to 4 times the previous value, the new time period will become. [AIEEE -2003]

1)10 hrs 2) 80 hrs 3) 40 hrs 4) 20 hrs

52. If the mass of earth were 2 times the present mass, the mass of the moon were half the present mass and the moon were revolving round the earth at the same present distance, the time period of revolution of the moon would be

1) 56 days 2) 28 days
3) $14\sqrt{2}$ days 4) 7 days

53. A satellite of mass 'm' revolves round the earth of mass 'M' in a circular orbit of radius'r' with an angular velocity 'ω'. If the angular velocity is ω/8 then the radius of the orbit will be

1) 4r 2) 2r
3) 8r 4) r

54. Two satellites P, Q are revolving around earth in different circular orbits. The velocity of P is twice the velocity of Q. If the height of P from earth's surface is 1600 km. The radius of orbit of Q is (radius of earth R =6400 km).

1) 1600 km 2) 20000 km
3) 32000 km 4) 40000 km

55. If an artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth, the height of the satellite above the surface of the earth is

1) 2R 2) R/2 3) R 4) R/4

56. The moon revolves round the earth 13 times in one year. If the ratio of sun-earth distance to earth-moon distance is 392, then the ratio of masses of sun and earth will be

1) 36 2) 356 3) 3.56×10^5 4) 1

57. A satellite is launched into a circular orbit of radius R around the earth. A second satellite is launched into an orbit of radius 1.01 R. The time period of the second satellite is larger than that of the first one by approximately

1) 0.5% 2) 1.5% 3) 1% 4) 3%

58. An astronaut orbitting in a spaceship round the earth has a centripetal acceleration of $2.45 m/s^2$. The height of spaceship from earth's surface is (R= radius of earth)

1) 3R 2) 2R 3) R 4) $R/2$

59. An electron revolves in Bohr's 1st orbit with frequency f.The frequency of the electron in Bohr's 2nd orbit is

1) f/8 2) 8f 3) 4f 4) f/4

ENERGY OF THE ORBITING SATELLITES

60. A satellite moves around the earth in a circular orbit with speed 'V'. If 'm' is mass of the satellite then its total energy is

1) $\frac{1}{2}mv^2$ 2) mv^2 3) $-\frac{1}{2}mv^2$ 4) $\frac{3}{2}mv^2$

61. Energy required to move a body of mass 'm' from an orbit of radius 2R to 3R is [AIEEE-2002]

1) $\frac{GMm}{12R}$ 2) $\frac{GMm}{3R^2}$

3) $\frac{GMm}{8R}$ 4) $\frac{GMm}{6R}$

62. The K.E. of a satellite in an orbit close to the surface of the earth is E. Its max K.E. so as to escape from the gravitational field of the earth is.

1) 2E 2) 4E

3) $2\sqrt{2}$ E 4) $\sqrt{2}$ E

63. Two satellites of masses 400 kg, 500 kg are revolving around earth in different circular orbits of radii r_1, r_2 such that their kinetic energies are equal. The ratio of r_1 to r_2 is

1) 4 : 5 2) 16 : 25

3) 5 : 4 4) 25 : 16

64. The self potential energy of a spherical shell of mass 'M' and radius 'R'

1) $\frac{-GM^2}{R}$ 2) $\frac{-GM^2}{2R}$

3) $\frac{-3}{5}\frac{GM^2}{R}$ 4) $\frac{-GM^2}{4R}$

65. A particle falls towards earth from infinity. The velocity with which it reaches earth's surface is.

1) v = 2gR 2) $v=\sqrt{2gR}$

3) $v=\sqrt{gR}$ 4) v = R/g

GEOSTATIONARY AND POLAR SATELLITES

66. The time period of a satellite very close to earth is 'T'. The time period of geo-synchronous satellite will be

1) $2\sqrt{2}(T)$ 2) $6\sqrt{6}(T)$

3) $7\sqrt{7}(T)$ 4) $\frac{1}{7\sqrt{7}}(T)$

67. The orbital speed of geostationary satellite is

1) 8km/sec from west to east
2) 11.2km/sec from east to west
3) 3.1km/sec from west to east
4) zero

68. Polar satellites go round the poles of earth in

1) South-east direction
2) north-west direction
3) east-west direction
4)north-south direction

WEIGTHLESSNESS

69. The time period of rotation of the earth around its axis so that the objects at the equator become weightless is nearly (g = $9.8m/s^2$, Radius of earth = 6400km.)

1) 64min 2) 74min

3) 84min 4) 94min

70. Masses 4 kg and 36 kg are 16 cm apart. The point where the gravitational field due to them is zero is

1) 6 cm from 4 kg mass
2) 4 cm from 4 kg mass
3) 1.8 cm from 36 kg mass
4) 9 cm from each mass

KEY

LEVEL-I

1)2	2)2	3)2	4)2	5) 4
6)4	7)1	8)2	9)2	10)2
11)1	12)1	13)3	14)3	15)3
16)3	17)2	18)1	19)4	20)1
21)1	22)1	23)3	24)2	25)2
26)2	27)3	28)1	29)4	30)2
31)1	32)4	33)2	34)2	35)4
36)2	37)1	38)1	39)3	40)3
41)2	42)4	43)4	44)1	45)3
46)4	47)1	48)1	49)2	50)4
51)3	52)3	53)1	54)3	55)3
56)3	57)2	58)3	59)1	60)3
61)1	62)1	63)1	64)3	65)2
66)3	67)3	68)4	69)3	70)2

HINTS

LEVEL-I

1. $\frac{dA}{dt} = \frac{L}{2m}$
2. $\frac{dA}{dt} = \text{constant}$
3. $T^2 \alpha r^3$
4. $Vr = \text{constant}$
5. $M = \frac{4}{3}\pi R^3 \rho$ and $T = 2\pi\sqrt{\frac{R^3}{GM}}$
6. $\frac{dA}{dt} = \frac{L}{2m}$
7. $F\alpha \frac{1}{R^n}$ but $F = mR\omega^2$
8. $F\alpha m_1 m_2$ but $m\alpha r^3 d$
9. $F = \frac{Gm_1 m_2}{r^2}$
10. $F_R = \sqrt{3}F$
11. $F\alpha m_1 m_2$
12. $\frac{dF}{dm} = 0 \Rightarrow \frac{d}{dm}(m)(M-m) = 0$
13. $F = \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$
14. $F_R = \sqrt{F_1^2 + F_2^2}$
15. $g\alpha \frac{1}{R^2}$
16. $g' = g\left(1 - \frac{d}{R}\right)$
17. $g \,\alpha \frac{M}{R^2}$
18. $g - R\omega^2 \cos^2\lambda = 0$
19. $R_{\max} = \frac{u^2}{g}$
20. $e\alpha g$ but $g\alpha \frac{1}{R^2}$
21. $F = \frac{GM'm}{x^2}$ but $\frac{M'}{x^3} = \frac{M}{R^3}$
22. $mg' = mg - mR\omega^2 \cos^2\lambda$
23. $g\alpha R\rho$
24. $g' = g - r\omega^2 \cos^2\lambda$
25. $x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1}$
26. $x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1}$
27. $E = E_1 + E_2 + ...$
28. $\vec{I}_1 = \frac{Gm_2}{d^2}$ and $\vec{I}_2 = -\frac{Gm_1}{d^2}$
29. $GPE = \frac{Gm_1 m_2}{r}$
30. $w = GPE_2 - GPE_1$
31. $\Delta GPE = \frac{mgh}{1 + \frac{h}{R}}$
32. $w = m(\Delta v) + \Delta KE$
33. $F = \frac{-du}{dr}$
34. $E = \frac{du}{dr}$
35. $L = \pi r$ but $V = \frac{-GM}{r}$
36. $w = GPE_2 - GPE_1$
37. $V\alpha\sqrt{gR}$
38. $V = \sqrt{\frac{2GM}{R}}$ but $M = \frac{4}{3}\pi R\rho$

39. $h = \frac{Rk^2}{1-k^2}$

40. $V = \sqrt{\frac{2GM}{R}}$

41. $V = \sqrt{V^2 - V_e^2}$

42. $V_e = \sqrt{2}V_0$

43. $|GPE| = KE$

44. $V_e \alpha R\sqrt{\rho}$

45. TE = constant

46. TE = constant

47. $V \alpha \frac{1}{\sqrt{r}}$

48. $V \alpha \frac{1}{\sqrt{R+h}}$

49. $V = \sqrt{\frac{GM}{r}}$

50. $T^2 \alpha r^3$

51. $T^2 \alpha r^3$

52. $T \alpha \sqrt{\frac{r^3}{M}}$

53. $\omega^2 \alpha \frac{1}{r^3}$

54. $V = \sqrt{\frac{GM}{R+h}}$

55. $V = \frac{1}{2}V_e$

56. $T^2 \alpha \frac{r^3}{M}$

57. $\frac{\Delta T}{t} \times 100 = \frac{3}{2}\frac{\Delta r}{r} \times 100$

58. $a = \frac{gR^2}{(R+h)^2}$

59. $f \,\alpha\, \frac{1}{T} \,\alpha\, \frac{1}{r^{3/2}}$ but $r \,\alpha\, n^2$

60. $\frac{KE}{TE} = -1$

61. $E = \frac{-GMm}{2r}$

62. $K' = 2K$

63. $KE = \frac{GMm}{2r}$

64. $\frac{-GM^2}{R}\left(\frac{3}{5}\right)$

65. $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$

66. $\frac{T'}{T} = \sqrt{\left(\frac{7R}{R}\right)^3}$

67. Conceptual

68. Conceptual

69. $T = 2\pi\sqrt{\frac{R}{g}}$

70. $x = \frac{d}{\sqrt{\frac{m_2}{m_1}+1}}$

LEVEL-II

KEPLER'S LAWS

1. A "double star" is a composite system of two stars rotating about their centre of mass under their mutual gravitational attraction. Let us consider such a "double star" which has two stars of masses 'm' and '2m' at a separation 'l'. If T is the time period of rotation about their entre of mass then.

1) $T = 2\pi\sqrt{\frac{l^3}{mG}}$ 2) $T = 2\pi\sqrt{\frac{l^3}{2mG}}$

3) $T = 2\pi\sqrt{\frac{l^3}{3mG}}$ 4) $T = 2\pi\sqrt{\frac{l^3}{4mG}}$

2. **The minimum and maximum distance of a satellite from the centre of earth is 2R and 4R respectively. Its maximum speed is**

1) $\sqrt{\frac{GM}{6R}}$ 2) $\sqrt{\frac{2GM}{3R}}$

3) $\sqrt{\frac{GM}{3R}}$ 4) $\sqrt{\frac{2GM}{5R}}$

3. **A satellite is revolving around a planet of mass 'm' in an elliptical orbit of semi major axis 'a'. The orbital velocity of the satellite at a distance 'r' from the focus will be**

1. $\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$ 2. $\sqrt{GM\left(\frac{1}{r}-\frac{2}{a}\right)}$

3. $\sqrt{GM\left(\frac{2}{r^2}-\frac{1}{a^2}\right)}$ 4. $\sqrt{GM\left(\frac{1}{r^2}-\frac{2}{a^2}\right)}$

4. **Two satellites S_1 and S_2 are revolving round a planet in coplanar and concentric circular orbits of radii R_1 and R_2 in the same direction respectively. Their respective periods of revolution are 1 hr. and 8 hr. The radius of the orbit of satellite S_1 is equal to 10^4km. Their relative speed when they are closest, in kmph is**

(2002 M)

1) $\frac{\pi}{2}\times 10^4$ 2) $\pi\times 10^4$

3) $2\pi\times 10^4$ 4) $4\pi\times 10^4$

5. **A binary star system consists of two stars of masses M_1 and M_2 revolving in circular orbits of radii R_1 and R_2 respectively. If their respective time periods are T_1 and T_2**

1) $T_1 > T_2$ if $R_1 > R_2$

2) $T_1 > T_2$ if $M_1 > M_2$

3) $T_1 = T_2$

4) $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$

LAW OF GRAVITATION

6. **Four particles of masses m, 2m, 3m and 4m are placed at the corners of a square of side length a. The gravitational force on a particle of mass m placed at the centre of the square is**

1) $4\sqrt{2}\frac{Gm^2}{a^2}$ 2) $\frac{3\sqrt{2}Gm^2}{a^2}$

3) $\frac{2\sqrt{2}Gm^2}{a^2}$ 4) $\frac{\sqrt{2}Gm^2}{a^2}$

7. **The gravitational P.E. of a rocket of mass 100 kg at a distance of 10^7 m from the earth's centre is –4 x 10^9J. The weight of the rocket at a distance of 10^9 m from the centre of the earth is**

1) 4 x 10^{-2} N 2) 4 x 10^{-9} N

3) 4 x 10^{-6} N 4) 4 x 10^{-3} N

8. **Three particles, each of mass 'm' are situated at the vertices of an equilateral triangle of side 'a'. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle should move in a circle while maintaining the original mutual separation 'a'. Then their time period of revolution is**

1) $2\pi\sqrt{\frac{a^2}{3Gm}}$ 2) $2\pi\sqrt{\frac{a^3}{3Gm}}$

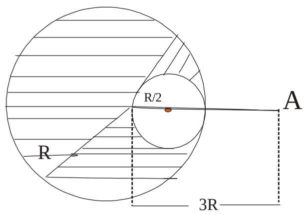
3) $2\pi\sqrt{\frac{3a^4}{Gm}}$ 4) $2\pi\sqrt{\frac{a^4}{Gm}}$

9. **Two particles each of mass 'm' are placed at A and C are such AC=BC=L. The gravitational force on the third particle placed at D at a distance L on the perpendicular bisector of the line AC is**

1) $\frac{Gm^2}{\sqrt{2}L^2}$ along BD 2) $\frac{Gm^2}{\sqrt{2}L^2}$ along DB

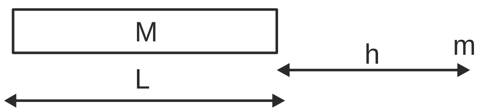
3) $\frac{Gm^2}{L^2}$ along AC 4) none of these

10. A solid sphere of uniform density and radius 'R' applies a gravitational force of attraction equal to F_1 on a particle placed at a distance 3R from the centre of the sphere. A spherical cavity of radius '$\frac{R}{2}$' is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle. The ratio $\frac{F_2}{F_1}$ is



1) 9/50 2) 41/50 3) 3/25 4) 22/25

11. A homogeneous bar of length L and mass M is at a distance 'h' from a point mass 'm' as shown. The force on 'm' is F. Then



1) $F=\frac{GMm}{(h+L)^2}$ 2) $F=\frac{GMm}{h^2}$

3) $F=\frac{GMm}{h(h+L)}$ 4) $F=\frac{GMm}{L^2}$

12. Two lead balls of masses m and 5m having radii R and 2R are separated by 12R. If they attract each other by gravitational force, the distance covered by small sphere before they touch each other is

1) 10 R 2) 7.5 R 3) 9 R 4) 2.5 R

13. Explorer- 38, a radio-activity research satellite of mass 200 kg circles the earth in an orbit of radius 3R/2, where R is the radius of the earth. Assuming the gravitational pull on a mass of 1 kg at the earth's surface to be 10 N, the pull on the satellite is

1) 889 N 2) 4500 N 3) 9000 N 4) None

14. Three uniform spheres of mass M and radius R earth are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the spheres due to the other two is

1) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ 2) $\frac{3}{2}\frac{GM^2}{R^2}$

3) $\frac{\sqrt{3}GM^2}{R^2}$ 4) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

15. Consider a particle of mass m suspended varticaly by a string at the equator. Let R and M denote the radius and the mass of the earth respectively. If ω is the angular velocity of earth's rotation about its own axis, the tension in the string is equal to

1) $G\frac{mM}{2R^2}$ 2) $G\frac{mM}{R^2}$

3) $G\frac{mM}{R^2}-m\omega^2R$ 4) $G\frac{mM}{R^2}+m\omega^2R$

ACCELERATION DUE TO GRAVITY AND ITS VARIATION WITH ALTITUDE, DEPTH AND ROTATION OF EARTH

16. The angular velocity of earth's rotation about its axis is 'ω'. An object weighed by a spring balance gives the same reading at the equator as at a height 'h' above the poles. The value of 'h' will be

1) $\frac{\omega^2R^2}{g}$ 2) $\frac{\omega^2R^2}{2g}$ 3) $\frac{2\omega^2R^2}{g}$ 4) $\frac{2\omega^2R^2}{3g}$

17. In the above problem, if the reading of the spring balance is same as that at depth 'd' below the earth's surface at poles. The value of 'd' will be

1) $\frac{\omega^2R^2}{g}$ 2) $\frac{\omega^2R^2}{2g}$

3) $\frac{2\omega^2R^2}{g}$ 4) $\frac{2\omega^2R^2}{3g}$

18. At a given place where acceleration due to gravity is $g\,m/\sec^2$, a sphere of lead of density $d\,kg/m^3$ is gently released in a column of liquid of density $\rho\,kg/m^3$. If $d > \rho$, the sphere will

1) fall vertically with an acceleration of $g\,m/\sec^2$
2) fall vertically with no acceleration
3) fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$
4) fall vertically with an acceleration $g\rho/d$

19. The radius of a planet is R. A satellite revolves around it in a circle of radius *r* with angular speed 'ω'. The accelaration due to gravity on planet's surface will be

1) $\frac{r^2\omega}{R}$ 2) $\frac{r^2\omega^2}{R}$
3) $\frac{r^3\omega^2}{R^2}$ 4) $\frac{r^2\omega^3}{R}$

20. If 'g' is acceleration due to gravity and 'R' is the radius of earth and angular speed of rotation of earth about its axis is made $\sqrt{\frac{2g}{5R}}$, then the weight of the body at the equator decreases by

1) 60 % 2) 50 %
3) 40 % 4) 75 %

21. An object weighs 10 N at the north pole of the earth. In a geostationary satellite distant 7 R from the centre of the earth (of radius R), the true weight and the apparent weight are

1) 0 N, 0 N 2) 0.2 N, 0
3) 0.2 N, 9.8 N 4) 0.2 N, 0.2 N

22. The radius of the planet is n times the radius of earth (R) . A satellite revolves around it in a circle of radius 4nR with angular velocity 'ω'. Then the acceleration due to gravity on planet's surface is

1) $R\omega^2$ 2) $16R\omega^2$
3) $32nR\omega^2$ 4) $64nR\omega^2$

GRAVITATIONAL POTENTIAL AND GRAVITATIONAL POTENTIAL ENERGY

23. Two identical particles each of mass 'm' start moving towards each other from rest from infinite separation under gravitational attraction. Their relative velocity of approach at separation 'r' is

1) $\sqrt{\frac{Gm}{r}}$ 2) $\sqrt{\frac{2Gm}{r}}$
3) $2\sqrt{\frac{Gm}{r}}$ 4) $\sqrt{\frac{Gm}{2r}}$

24. The gravitational intensity in a region is $10\left(\hat{i}-\hat{j}\right)N/Kg$. The work done by the gravitational force to shift slowly a particle of mass 1 Kg from point (1m, 1m) to a point (2m, -2m) is

1) 10J 2) –10J 3) –40J 4) +40J

25. A point $P(\sqrt{3}R, 0, 0)$ lies on the axis of a ring of a mass 'M' and radius 'R'. The ring is located in y-z plane with its centre at origin 'O'. A small particle of mass 'm' starts from 'P' and reaches 'O' under gravitational attraction only. Its speed at 'O' will be

1) $\sqrt{\frac{GM}{R}}$ 2) $\sqrt{\frac{Gm}{R}}$
3) $\sqrt{\frac{GM}{\sqrt{2}R}}$ 4) $\sqrt{\frac{Gm}{\sqrt{2}R}}$

26. The gravitational field in X-direction due to some mass distribution is $E=\frac{k}{x^3}$, where k is a constant. assuming the gravitational potential to be zero at infinity, its value at a distance x will be

1) $\frac{k}{x}$ 2) $\frac{k}{2x}$ 3) $\frac{k}{x^2}$ 4) $\frac{k}{2x^2}$

27. The magnitude of the gravitational force between a particle of mass m_1 **and another particle of mass m_2 is F = Gm_1m_2/x^2. The work required to increase the separation of the particles from x = x_1 to x= (x_1 + d) is**

1) $\frac{Gm_1m_2x_1}{d(x_1+d)}$ 2) $\frac{Gm_1m_2d}{x_1(x_1+d)}$

3) $\frac{Gm_1m_2x_1^2}{d(x_1+d)}$ 4) $\frac{Gm_1m_2d^2}{x_1(x_1+d)}$

28. If d is the distance between the centres of the earth of mass M_1 and moon of mass M_2, then the velocity with which a body should be projected from the mid point of the line joining the earth a

1) $\sqrt{\frac{G(M_1+M_2)}{d}}$ 2) $\sqrt{\frac{G(M_1+M_2)}{2d}}$

3) $\sqrt{\frac{2G(M_1+M_2)}{d}}$ 4) $\sqrt{\frac{4G(M_1+M_2)}{d}}$

29. Three particles of equal mass 'm' are situated at the vertices of an equilateral triangle of side 'L'. The work done in increasing the side of the triangle to 2L will be

1) $\frac{2G^2m}{2L}$ 2) $\frac{Gm^2}{2L}$

3) $\frac{3Gm^2}{2L}$ 4) $\frac{3Gm^2}{L}$

30. The workdone in slowly lifting a body from earth's surface to a height R(radius of earth) is equal to two times the workdone in lifting the same body from earth's surface to a height *h*. Here *h* is equal to

1) R/4 2) R/3 3) R/6 4) R/2

31. The gravitational field in a region is given by $\vec{E}=\left(4\hat{i}+\hat{j}\right)$ N/kg .Workdone by this field is zero when a particle is moved along the line

1) $y+4x=2$ 2) $4y+x=6$

3) $x+y=5$ 4) all of these

32. Two bodies of masses m and M are placed a distance d apart. The gravitational potential at the position were the gravitational field due to them is zero is

1) $V=\frac{-G}{d}(m+M)$

2) $V=\frac{-Gm}{d}$

3) $V=\frac{-GM}{d}$

4) $V=\frac{-G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$

33. The value of the acceleration due to gravity at the surface of the earth of radius R is g. It decreases by 10% at height h above the surface of the earth. The gravitational potential at this height is

1) $-\frac{gR}{\sqrt{10}}$ 2) $-\frac{2gR}{\sqrt{10}}$

3) $-\frac{3gR}{\sqrt{10}}$ 4) $-\frac{4gR}{\sqrt{10}}$

34. A satellite of mass m is moving in a circular orbit of radius R above the surface of a planet of mass M and radius R. The amount of work done to shift the satellite to a higher orbit of radius 2R from the surface of the planet is (here g is the acceleration due to gravity on planet's surface)

1) mgR

2) $\frac{mgR}{6}$

3) $\frac{m(MgR)}{(M+m)}$

4) $\frac{mMgR}{6(M+m)}$

GRAVITATIONAL FIELD INTENSITY

35. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are E_1 and E_2 respectively. Then:

1) $\frac{E_1}{E_2}=\frac{r_1}{r_2}$ if $r_1 <$ R and $r_2 <$ R

2) $\frac{E_1}{E_2}=\frac{r_2^2}{r_1^2}$ if $r_1 <$ R and $r_2 <$ R

3) $\frac{E_1}{E_2}=\frac{r_1^3}{r_2^3}$ if $r_1 <$ R and $r_2 <$ R

4) $\frac{E_1}{E_2}=\frac{r_1^2}{r_2^2}$ if $r_1 <$ R and $r_2 <$ R

36. Two masses 90 kg and 160 kg are 5 m apart. The gravitational field intensity at a point 3m from 90 kg and 4 m from 160 kg is

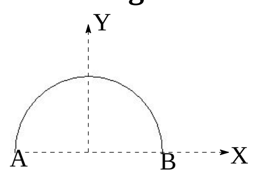
1) 10 G 2) 5 G 3) $5\sqrt{2}G$ 4) $10\sqrt{2}G$

37. If the gravitational field intensity at a point is $\frac{GM}{r^{2.5}}$, then the potential at a distance 'r' is

1) $\frac{2GM}{3r^{1.5}}$ 2) $\frac{-GM}{r^{3.5}}$

3) $-\frac{2GM}{3r^{1.5}}$ 4) $\frac{GM}{r^{3.5}}$

38. Gravitational field intensity at the centre of the semi circle formed by a thin wire AB of mass 'm' and length 'L' is



1) $\frac{Gm^2}{L^2}(\hat{i})$ 2) $\frac{Gm^2}{\pi L^2}(\hat{j})$

3) $\frac{2\pi Gm}{L^2}(\hat{i})$ 4) $\frac{2\pi Gm}{L^2}(\hat{j})$

ESCAPE SPEED

39. If 'V_e' is the escape velocity of a body from a planet of mass 'M' and radius 'R'. Then the velocity of the satellite revolving at height 'h' from the surface of the planet will be

1) $V_e\sqrt{\frac{R}{R+h}}$ 2) $V_e\sqrt{\frac{2R}{R+h}}$

3) $V_e\sqrt{\frac{R+h}{R}}$ 4) $V_e\sqrt{\frac{R}{2(R+h)}}$

40. The escape velocity from a planet is 'V_e'. A tunnel is dug along along a diameter of the planet and a small body is dropped into it. The sped of the body at the centre of the planet will be

1) V_e 2) $\frac{V_e}{2}$

3) $2V_e$ 4) $\frac{V_e}{\sqrt{2}}$

41. A boy can jump to a height 'h' on the ground level. What should be the radius of a sphere of density δ such that on jumping on it, he escapes out of the gravitational field of the sphere?

1) $\sqrt{\frac{4\pi G\delta}{3gh}}$ 2) $\sqrt{\frac{4\pi gh}{3G\delta}}$

3) $\sqrt{\frac{3gh}{4\pi G\delta}}$ 4) $\sqrt{\frac{3G\delta}{4\pi gh}}$

42. Three particles are projected vertically upward from a point on the surface of earth with velocites $\sqrt{\frac{2gR}{3}}, \sqrt{gR}$ and $\sqrt{\frac{4}{3}gR}$ respectively. The ratio of maximum heights attained by them are

1) $4:2:1$ 2) $1:2:4$

3) $2:3:4$ 4) $1:4:9$

EARTH SATELLITES

43. A satellite is orbitting at a height of $h = 10R$, where r is the radius of the earth from the surface of the earth for which orbital velocity is $\sqrt{\frac{11gR^2}{(R+h)}}$. In which curve the satellite orbits the earth ?

1) Circle 2) Hyperbola
3) Falls on the earth 4) Ellipse

44. A satellite is revolving round the earth with orbital speed V_0. If it stops suddenly, the speed with which it will strike the surface of earth would be:(V_e = escape velocity of a particle on earth's surface)

1) $\frac{V_e^2}{V_0}$ 2) V_0
3) $\sqrt{V_e^2 - V_0^2}$ 4) $\sqrt{V_e^2 - 2V_0^2}$

ENERGY OF THE ORBITTING SATELLITE

45. Two statellites of same mass are launched in the same orbit around earth so as to rotate opposite to each other. If they collide inelastically and stick together as wreckage, the total energy of the system just after collision is

1) $\frac{-2GMm}{r}$ 2) $\frac{-GMm}{r}$
3) $\frac{GMm}{2r}$ 4) $\frac{GMm}{4r}$

46. A rocket is fired vertically from mars surface with 2km/sec. If 20% of its initial kinetic energy is lost due to martian atmospheric resistance, then the maximum height that the rocket reaches is (mass of mars $= 6.4 \times 10^{23} kg$; radius of mars =3400km)

1) 2000 km 2) 1500 km
3) 1000 km 4) 500 km

47. The ratio of the energy required to raise a satellite upto a height R (radius of earth) from the surface of earth to that required to put it into orbit there is

1) 1:1 2) 8:1 3) 4:1 4) 2:3

48. An object is projected with a velocity $\sqrt{\frac{8gR}{3}}$ from earth. The velocity of the object at the maximum height will be

1) $\sqrt{\frac{2gR}{3}}$ 2) $\sqrt{\frac{8r}{3}}$
3) $\sqrt{2gR}$ 4) zero

49. The workdone in slowly lifting a body from earth's surface to a height R(radius of earth) is equal to 2.5 times the workdone in lifting the same body from earth's surface to a height *h*. Here *h* is equal to

1) $\frac{R}{4}$ 2) $\frac{R}{3}$ 3) $\frac{R}{6}$ 4) $\frac{R}{2}$

50. A particle is projected upward from the surface of earth (radius=R) with a speed equal to the orbital speed of a satellite near the earth's surface. The height to which it would rise is

1) $\sqrt{2}R$ 2) $\frac{R}{\sqrt{2}}$ 3) R 4) $2R$

51. An artificial satellite moving in circular orbit around the earth has a total (kinetic + potential) energy E_0,. Its potential energy and kinetic energy respectively are

1) $2E_0$ and $-2E_0$ 2) $-2E_0$ and $3E_0$
3) $2E_0$ and $-E_0$ 4) $-2E_0$ and $-E_0$

52. A stone is dropped from a height equal to nR, where R is the radius of the earth, from the surface of the earth. The velocity of the stone on reaching the surface of the earth is

1) $\sqrt{\frac{2g(n+1)R}{n}}$ 2) $\sqrt{\frac{2gR}{n+1}}$
3) $\sqrt{\frac{2gnR}{n+1}}$ 4) $\sqrt{2gnR}$

53. Two particles of same mass fall onto the earth's surface, one from infinity and other from an altitude 3R. The ratio of velocity on eaching the earth is

1) $2:3$ 2) $3:2$

3) $2:\sqrt{3}$ 4) 1 : 2

KEY

LEVEL-II

1) 3	2) 2	3) 1	4) 2	5) 3
6) 1	7) 1	8) 2	9) 2	10) 2
11) 3	12) 2	13) 1	14) 1	15) 3
16) 2	17) 1	18) 3	19) 3	20) 3
21) 2	22) 4	23) 3	24) 4	25) 1
26) 4	27) 2	28) 4	29) 3	30) 2
31) 1	32) 4	33) 3	34) 2	35) 1
36) 4	37) 1	38) 4	39) 4	40) 4
41) 3	42) 2	43) 1	44) 4	45) 1
46) 4	47) 4	48) 1	49) 1	50) 3
51) 3	52) 3	53) 3		

HINTS

LEVEL-II

1. consider origin at partical of mass 2m,we have centre of mass is l/3 and 2l/3 from m. $m\left(\frac{2l}{3}\right)\omega^2=\frac{Gm(2m)}{l^2}$ then find T

2. Vr = constand and TE = constant

3. Elliptical orbit is $E=-\frac{GMm}{2a}$, remains constant

$\therefore$ According to law of conservation of energy

$\frac{1}{2}mV^2-\frac{GMm}{r}=-\frac{GMm}{2a}$ (at position 'r')

$\Rightarrow\frac{1}{2}mV^2=\frac{GMm}{r}-\frac{GMm}{2a}$

$\Rightarrow V=\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$

4. $\frac{T_1^2}{T_2^2}=\frac{R_1^3}{R_2^3}, V_{O_1}=R_1\omega_1, V_{O_2}=R_2\omega_2$

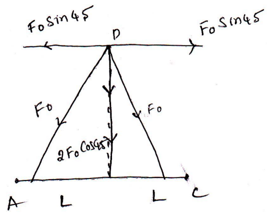
5. The two stars rotate about common center of mass with the same angular velocity.

6. Find individual force and then resultant

7. $GPE=\frac{-GMm}{R}$ and $g=\frac{GM}{R^2}$

8. $mr\omega^2=\sqrt{3}\frac{Gm^2}{a^2}$ and $r=\frac{a}{\sqrt{3}}$

9 :



$F_0=\frac{G(m)(m)}{\left(\sqrt{2}L\right)^2}=\frac{Gm^2}{2L^2}$

$\therefore$ Resultant force $=2F_0Cos45^0$

$=2\left(\frac{Gm^2}{2L^2}\right)Cos45^0=\frac{Gm^2}{\sqrt{2}L^2}$

10. From super position principle, $F_1=F_r+F_c$

Here F_r = force due to remaining part = F_2

F_c =force due to mass on the cavity

$F_1=\frac{GMm}{2}=\frac{GMm}{9R^2}$; $F_c=\frac{G\left(\frac{M}{8}\right)m}{\left(\frac{5}{2}R\right)^2}=\frac{GMm}{50R^2}$

$\Rightarrow F_2=F_1-F_c=\frac{GMm}{9R^2}-\frac{GMm}{50R^2}=\frac{41GMm}{450R^2}$

$\Rightarrow\frac{F_2}{F_1}=\frac{41}{50}$

11. F= $\int_h^{L+h}\frac{GMm}{x^2L}dx$

12. Effective distance =9R
Distance travelled by Smaller mass= x

$$x=\left(\frac{m_2}{m_1+m_2}\right)(9R)$$

13. $\frac{F_1}{F_2}=\frac{GM.1}{R^2}\times\frac{\left(\frac{3R}{2}\right)^2}{GM.200}$

14. Force between any two spheres will be

$$F=\frac{G(M)(M)}{(2R)^2}=\frac{GM^2}{4R^2}$$

The two forces of equal magnitude F are acting at angle 60^0 on any of the sphere

$$F_{net}=\sqrt{F^2+F^2+2(F)(F)\cos 60^0}=\sqrt{3}F$$

15. T = gravitational force - centrifugal force

16. $mg-m\omega^2R=mg\left(1-\frac{2h}{R}\right)$

$$\Rightarrow \omega^2R=\frac{2gh}{R}\Rightarrow h=\frac{\omega^2R^2}{2g}$$

17. $mg-m\omega^2R=mg\left(1-\frac{d}{R}\right)$

$$\Rightarrow \omega^2R=\frac{gd}{R}\quad\Rightarrow d=\frac{\omega^2R^2}{g}$$

18. $a=\frac{mg-F_B}{m}=\frac{\frac{4}{3}\pi r^3dg-\frac{4}{3}\pi r^3\rho g}{\frac{4}{3}\pi r^3 d}$

19. Let M be the mass of the planet and m the mass of the satellite. Then $mrw^2=\frac{GMm}{r^2}$

20. $g'=g-R\omega^2$

21. $g\alpha\frac{1}{R^2}$

22. $g\alpha\frac{1}{R^2}$ and $F=mr\omega^2$

23. From conservation of energy

$$\frac{GMm}{r}=\frac{1}{2}mv^2+\frac{1}{2}mv^2$$

relative velocity = $2\times v$

24. $F_g=m\overrightarrow{E_g}=1\left(10\left(\hat{i}-\hat{j}\right)\right)=10\left(\hat{i}-\hat{j}\right)N=10\hat{i}-10\hat{j}$

$$\vec{S}=\left(2\hat{i}-2\hat{j}\right)-\left(\hat{i}-\hat{j}\right)=\hat{i}-3\hat{j}m$$

Work done $=\overrightarrow{F_g}.\vec{S}=\left(10\hat{i}-10\hat{j}\right).\left(\hat{i}-3\hat{j}\right)$

$$=10(1)+(-10)(-3)$$

25. According to law of conservation of energy

$$\frac{1}{2}mV^2=m(\Delta V)$$

$$\Rightarrow \frac{1}{2}mV^2=m\left[\frac{-GM}{2R}-\left(-\frac{GM}{R}\right)\right]$$

$$\Rightarrow \frac{1}{2}mV^2=m\left(\frac{GM}{2R}\right)\qquad\Rightarrow V=\sqrt{\frac{GM}{R}}$$

26. $V=-\int Edx$

27. W= $\int_{x_1}^{x_1+d}Fdx=\int_{x_1}^{x_1+d}\frac{Gm_1m_2}{x^2}dx$

28. Using law of conservation of energy
Kinetic energy minimum = – potential

$$PE\ at\ mid\ \text{point}=\frac{-2GMm}{d}(M_1+M_2)$$

$$\Rightarrow \frac{1}{2}mV_e^2=\frac{2Gm}{d}(M_1+M_2)$$

29: Initial potential energy, $U_i=\frac{-3Gm^2}{L}$

Final potential energy, $U_f=-\frac{3Gm^2}{2L}$

Work done, $W=U_f-U_i$

$$=-\frac{3Gm^2}{2L}-\left(-\frac{3Gm^2}{L}\right)=\frac{3Gm^2}{2L}$$

30. Workdone = increase in gravitational potential energy $\left(\Delta v = \frac{mgh}{1+\frac{h}{R}}\right)$

$w_1 = \frac{mgR}{1+\frac{R}{R}} = \frac{mgR}{2}$; $w_2 = \frac{mgh}{1+\frac{h}{R}}$; $W_1 = 2W_2$

31. Workdone will be zero when displacement is perpendicular to the field. The field makes an angle $\theta_1 = \tan^{-1}\left(\frac{1}{4}\right)$ with positive x- axis. The line $y+4x=2$ makes an angle $\theta_2 = \tan^{-1}(-4)$ with positive x- axis

32. If net gravitational field of p becomes zero means

$\frac{Gm}{x^2} = \frac{GM}{(d-x)^2} \Rightarrow x = \frac{(\sqrt{m})d}{\sqrt{m}+\sqrt{M}}$ and

$d - x = \frac{(\sqrt{M})d}{\sqrt{m}+\sqrt{M}}$

Gravitational potential at

$$\frac{-\mathrm{Gm}}{\frac{(\sqrt{\mathrm{m}})\mathrm{d}}{\sqrt{\mathrm{m}}+\sqrt{\mathrm{M}}}} + \frac{-\mathrm{Gm}}{\frac{(\sqrt{\mathrm{M}})\mathrm{d}}{\sqrt{\mathrm{m}}+\sqrt{\mathrm{M}}}} = -\frac{\mathrm{G}}{\mathrm{d}}\left(\sqrt{\mathrm{m}}+\sqrt{\mathrm{M}}\right)^2$$

33. $g \alpha \frac{1}{R^2}$ and $V = \frac{-GM}{r}$

34. $W = \frac{-GMm}{2R+R} - \left(-\frac{GMm}{R+R}\right)$

35 : If $r \leq R$, then $E = \frac{GM}{R^3}(r) \Rightarrow E \propto r$

$\Rightarrow \frac{E_1}{E_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

If $r \geq R$, then $E = \frac{GM}{r^2} \Rightarrow E \propto \frac{1}{r^2}$

$\Rightarrow \frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

36. $E_R = \sqrt{E_1^2 + E_2^2}$

37. $V = -\int E dr$

38. $\lambda = \frac{m}{L}$; $L = \pi r$

$dm = \lambda dl = \lambda(r d\theta)$

$E = \frac{G\lambda}{r}\left[\int_0^{\pi} \cos\theta d\theta \hat{i} + \int_0^{\pi} \sin\theta d\theta \hat{j}\right]$

39. $V_e = \sqrt{\frac{2GM}{R}}$ $\quad V = \sqrt{\frac{GM}{R+h}}$

$\Rightarrow \frac{V}{V_e} = \sqrt{\frac{R}{2(R+h)}} \Rightarrow V = V_e\sqrt{\frac{R}{2(R+h)}}$

40. $V_e = \sqrt{2gR}$

According to the law of conservation of energy

$U_c + \frac{1}{2}mV^2 = U_s \Rightarrow \frac{1}{2}mV^2 = U_s - U_c = m(V_s - V_c)$

$\Rightarrow \frac{1}{2}mV^2 = m\left[-\frac{GM}{R} - \left(-\frac{3GM}{2R}\right)\right]$

$\Rightarrow \frac{1}{2}mV^2 = m\left(\frac{GM}{2R}\right) \quad \Rightarrow V = \sqrt{\frac{GM}{R}} = \frac{V_e}{\sqrt{2}}$

$\left[\because V = \sqrt{gR}\right]$

41. Velocity of the body,

$V = \sqrt{2gh} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{8\pi GR^2}{3}\delta}$

$\Rightarrow \sqrt{2gh} = R\sqrt{\frac{8\pi G\delta}{3}} \Rightarrow R = \sqrt{\frac{3gh}{4\pi G\delta}}$

42. $h = \frac{Rk^2}{1-k^2}$

43. Closed orbit at a height

44. $\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{r}$

45 $TE = \frac{G(M)2m}{r}$

46. $\frac{80}{100}\frac{1}{2}mv^2 = \frac{-GMm}{R+h} - \left(\frac{-GMm}{R}\right)$

47. $\Delta E_1 = \frac{-GMm}{2R} + \frac{GMm}{R} = \frac{GMm}{2R}$

$\Delta E_2 = \frac{-GMm}{2(2R)} + \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R}$

$\Rightarrow \frac{\Delta E_1}{\Delta E_2} = \frac{2}{3}$

48. $V^1 = \sqrt{V^2 - V_e^2}$

49. $w = \frac{mgh}{1+\frac{h}{R}}$

50. $V = V_0 = \frac{V_e}{\sqrt{2}},$

51. $K.E = -E_{o,} P.E = 2E_o$

52. $\frac{1}{2}mv^2 = \frac{(mg)Rh}{R+h} = \frac{(mg)R(nR)}{(R+nR)}$

53. TE = constant

LEVEL-III

1. A point mass is orbiting a singnificant mass M lying at the focus of the elliptical orbit having major and minor axes given by 2a and 2b respectively. Let r be the distance between the mass M and the end point of major axis. Velocity of the particle can be given as

1) $\frac{ab}{r}\sqrt{\frac{GM}{a^3}}$

2) $\frac{ab}{r}\sqrt{\frac{GM}{b^3}}$

3) $\frac{ab}{2r}\sqrt{\frac{GM}{r^3}}$

4) $\frac{2ab}{r}\sqrt{\frac{GM}{\left(\frac{a+b}{2}\right)^3}}$

2. A binary star has stars of masses m and nm (where n is a numerical factor) having separation of their centres as d. If these stars revolve because of gravitational force of each other

a) The period of revolution is given by $\frac{2\pi r^{3/2}}{\left(\frac{Gnm^2}{(n+1)m}\right)^{\frac{1}{2}}}$

b) The period of revolution is given by $\frac{2\pi r^{\frac{1}{2}}}{\left(\frac{G(n+1)m}{nm^2}\right)^{1/2}}$

c) Reduced mass is $\frac{nm^2}{m(1+n)}$

d) Reduced mass is $\frac{mn^2}{n(1+m)}$

1) a, c 2) b, d 3) c, d 4) a, b

3. A planet of mass m revolves in elliptical orbit around the sun so that its maximum and minimum distances from the sun are equal to r_a and r_p respectively. Find the angular momentum of this planet relative to the sun

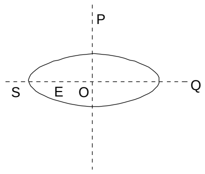
1) $L = m\sqrt{\frac{GMr_pr_a}{(r_p+r_a)}}$

2) $L = m\sqrt{\frac{2GMr_pr_a}{(r_p+r_a)}}$

3) $L = M\sqrt{\frac{Gmr_pr_a}{(r_p+r_a)}}$

4) $L = M\sqrt{\frac{(r_p+r_a)}{Gmr_pr_a}}$

4. **A satellite moving in elliptical orbit around earth as shown the minimum and maximum distance of trhe satellite from earth are 3 units and 5 units respectively. The distance of satellite from earth when it is at 'P' is —— (units)**



1) 4 2) 3 3) 3.75 4) 6

5. **The longest and the shortest distance of our planet from sun is R_1 and R_2. Distance from sun when it is normal to major axis of orbit is**

1) $\frac{R_1+R_2}{2}$ 2) $\sqrt{\frac{R^2{}_1+R^2{}_2}{2}}$

3) $\frac{R_1R_2}{R_1+R_2}$ 4) $\frac{2R_1R_2}{R_1+R_2}$

6. **A satellite is orbiting just above the surface of a planet of average density D with period T. If G is the universal gravitational constant, the quantity $\frac{3\pi}{G}$ is equal to**

1) T^2D 2) $3\pi T^2D$ 3) $3\pi D^2T$ 4) D^2T

7. **A planet revolve s around sun in an elliptical orbit of eccentricity 'e'. If 'T' is the time period of the planet then the time spent by the planet between the ends of the minor axis close to sun is**

1) $T\left(\frac{1}{4}-\frac{e}{2\Pi}\right)$ 2) $\frac{Te}{\Pi}$

3) $\left(\frac{e}{\Pi}-1\right)$ 4) $\frac{\Pi T}{e}$

8. **An artificial satellite revolves around earth in circular orbit of redius r with time period of orbit T. The satellite is made to stop in the orbit which makes it fall onto earth. Time of fall of the satellite onto earth is given by**

1) $\sqrt{3}\frac{T}{6}$ 2) $\frac{\sqrt{2}}{8}T$ 3) $\frac{T}{\sqrt{3}}$ 4) $\sqrt{\frac{2}{3}}\frac{T}{\pi}$

For problems 9, 10 and 11

The minimum and maximum distances of a satellite from sun are 2R and 4R, respecitvely, where R is the radius of earth and M is the mass of the sun.

9. **The length of the major axis of the orbit is**

1) 3R 2) 2R 3) $\frac{4}{3}R$ 4) 6R

10. **The minimum and maximum speeds**

1) $\sqrt{\frac{GM}{9R}}$ 2) $\sqrt{\frac{2GM}{9R}}$

3) $\sqrt{\frac{GM}{6R}},\sqrt{\frac{2GM}{3R}}$ 4) $\sqrt{\frac{GM}{R}},\sqrt{\frac{5GM}{2R}}$

11. **Radius of curvature at the point of minimum distance is**

1) $\frac{8R}{3}$ 2) $\frac{5R}{3}$ 3) $\frac{4R}{3}$ 4) $\frac{7R}{3}$

12. **The two planets with radii R_1, R_2 have densities ρ_1, ρ_2 and atmospheric pressures P_1 and P_2 respectively. Therefore the ratio of masses of their atmospheres, neglecting variation of g and ρ within the limits of atmosphere, is**

1) $\frac{P_1R_2\rho_1}{P_2R_1\rho_2}$ 2) $\frac{P_1R_2\rho_2}{P_2R_1\rho_1}$

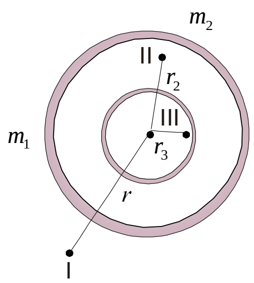
3) $\frac{P_1R_1\rho_1}{P_2R_2\rho_2}$ 4) $\frac{P_1R_1\rho_2}{P_2R_2\rho_1}$

13. **A homogeneous spherical heavenly body has a uniform and very narrow frictionless duct along its diameter. Let mass of the body be M and diameter be D. A point mass m moves smoothly inside the duct. Force exerted on this mass when it is at a distance s from the centre of the body is**

1) $\frac{GMm}{s^2}$ 2) $\frac{\pi GMm}{(D/2)^3}s$

3) $\frac{8GMm}{D^3}s$ 4) $\frac{GMm}{(R-s)^2}$.

14. **Two concentric shells of different masses m_1 and m_2 are having a sliding particle of mass m. Thie forces on the particle at position I, II and III are**



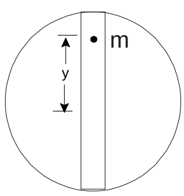
1) $0, \frac{Gm_1}{r_2^2}, \frac{G(m_1+m_2)m}{r_1^2}$

2) $\frac{Gm_2}{r_2^2}, 0, \frac{Gm_1}{r_1^2}$

3) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{Gm_2}{r_2^2}, 0$

4) $\frac{G(m_1+m_2)m}{r_1^2}, \frac{G(m_1)}{r_2^2}, 0$

15. **Suppose a vertical tunnel is dug along the diameter of earth assumed to be a sphere of uniform mass having density ρ. If a body of mass m is thrown in this tunnel, its acceleration at a distance y from the centre is given by**



1) $\frac{4\pi}{3} G\rho y m$ 2) $\frac{3}{4}\pi G\rho y$

3) $\frac{4}{3}\pi\rho y$ 4) $\frac{4}{3}\pi G\rho y$

16. **The pressure caused by gravitational pull inside earth at a distance a measured from its centre, when is mass, density and radius are M, ρ and R respectively , is given by**

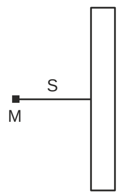
1) $\frac{8}{3}\frac{GM^2}{\pi R^3}\left(1-\frac{a^2}{R^2}\right)$

2) $\frac{3}{8}GM^2\left(1-\frac{a^2}{R^2}\right)$

3) $\frac{3}{8}\frac{GM^2}{\pi R^4}\left(1-\left(\frac{a}{R}\right)^2\right)$

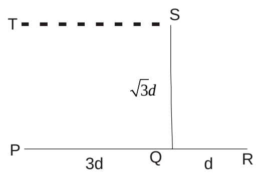
4) $\frac{8}{3}\frac{GM^2}{R^3}\left(1-\frac{a}{R}\right)$

17. **A point mass M is at a distance S from an infinitely long and thin rod of density D. If G is the gravitational constant then gravitational force between the point mass and the rod is**



1) $2\frac{MGD}{S}$ 2) $\frac{MGD}{S}$ 3) $\frac{MGD}{2S}$ 4) $\frac{2}{3}\frac{MGD}{S}$

18. **Three particles P, Q and R are placed as per given figure. Masses of P, Q and R are $\sqrt{3}m$, $\sqrt{3}$ m and m respectively. The gravitational force on a fourth particle S of mass m is equal to :**



1) $\frac{\sqrt{3}GM^2}{2d^2}$ in ST direction only

2) $\frac{\sqrt{3}GM^2}{2d^2}$ is SQ direction and $\frac{\sqrt{3}GM^2}{2d^2}$ in SU direction

3) $\frac{\sqrt{3}GM^2}{2d^2}$ is SQ direction only

4) $\frac{\sqrt{3}GM^2}{2d^2}$ in SQ direction and $\frac{\sqrt{3}GM^2}{2d^2}$ in ST direction

19. **A cavity of radius $\frac{R}{2}$ is made inside a solid sphere of radius R. The centre of the cavity is located at a distance $\frac{R}{2}$ from the centre of sphere. The gravitational force on a particle of mass m at a distance $\frac{R}{2}$ from the centre of the sphere on the line joining both the cetntres of sphere and cavity is (opposite to the centre of cavity) [here $g = \frac{GM}{R^2}$ where M is the mass of the sphere].**

1) $\frac{mg}{2}$ 2) $\frac{3mg}{8}$

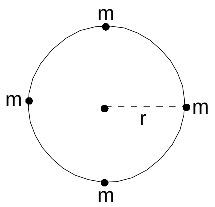
3) $\frac{mg}{16}$ 4) $\frac{mg}{4}$

20. **Two equal masses each 'm' are hung from a balance whose scale pans differ in vertical height by 'h'. the error in weighing is**

1) $\pi G\rho mh$ 2) $\frac{1}{3}G\rho mh$

3) $\frac{8}{3}\pi G\rho mh$ 4) $\frac{4}{3}\pi G\rho mh$

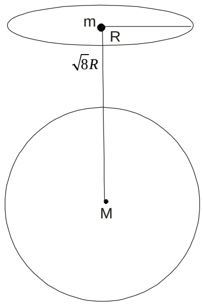
21. **Four masses 'm' each are orbiting in a circle of radius 'r' in the same direction under gravitational force. Velocity of each particle is**



1) $\sqrt{\frac{Gm}{r}\frac{\left(1+2\sqrt{2}\right)}{2}}$ 2) $\sqrt{\frac{Gm}{r}}$

3) $\sqrt{\frac{Gm}{r}\left(1+2\sqrt{2}\right)}$ 4) $\sqrt{\frac{Gm}{2r}\left(\frac{1+2\sqrt{2}}{2}\right)}$

22. **The centres of a ring of mass m and a sphere of mass M of equal radius R, are at a distance $\sqrt{8}$ R apart as shown in fig. The force of attraction between the ring and the sphere is**



1) $\frac{2\sqrt{2}}{27}\frac{GmM}{R^2}$ 2) $\frac{GmM}{8R^2}$

3) $\frac{GmM}{9R^2}$ 4) $\frac{\sqrt{2}}{9}\frac{GmM}{9R^2}$

23. **A spherical hollow is made in a lead sphere of radius R such that its surface touches the outside surface of the lead sphere and passes through the centre. The mass of the lead sphere before hollowing was M. The force of attraction that this sphere would exert on a particle of mass m which lies at a distance d from the centre of the lead sphere on the straight line joining the centres of the sphere and the hollow is**

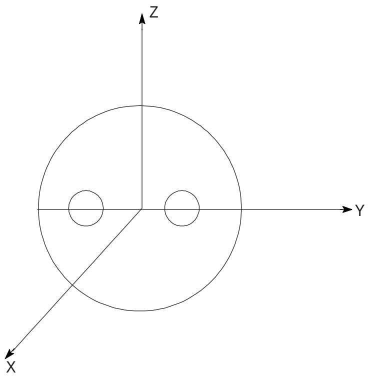
1) $\frac{GMm}{d^2}$

2) $\frac{GMm}{d^2}\left[1-\frac{1}{8\left(1-\frac{R}{2d}\right)^2}\right]$

3) $\frac{GMm}{d^2}\left[1+\frac{1}{8\left(1-\frac{R}{2d}\right)^2}\right]$

4) $\frac{GMm}{8d^2}$

24. **A solid sphere of uniform density and mass M has radius 4 metre. Its centre is at the origin of the coordinate system. Two spheres of radii 1 m are taken out so that their centres are at $P(0,-2,0)$ and $Q(0,2,0)$ respectively. This leaves two spherical cavities. What is the gravitational field at the origin of the coordinate axes ?**



1) $\frac{31GM}{1024}$ 2) $\frac{GM}{1024}$

3) $31GM$ 4) 0

25. **In the above question, what is the gravitational field at the centre of the cavities ?**

1) $\frac{31GM}{1024}$ 2) $\frac{GM}{1024}$

3) $31GM$ 4) 0

26. **In the above question, what is the gravitational potential at any point on the circle $x^2+z^2=6$?**

1) $-\frac{GM}{6}$

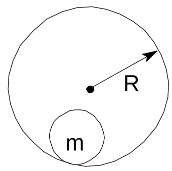
2) $-\frac{GM}{64\sqrt{10}}$

3) $-\frac{GM}{2}\left[\frac{1}{3}-\frac{1}{32\sqrt{10}}\right]$

4) $-\frac{GM}{2}\left[\frac{1}{3}+\frac{1}{32\sqrt{10}}\right]$

For problems: 27, 28 and 29

A solid sphere of mass 'm' and radius 'r' is placed inside a hollow thin spherical shell of mass M and radius 'R' as shown in the below. A particle of mass m^1 is placed on the line joining the two centres at a distance 'x' from the point of contact of sphere and shell. The magnitude of resultant gravitational force on this particle due to sphere and shell is



27. $r<x<2r$

1) $F=\frac{Gmm^1(2r-x)}{2r^3}$

2) $\frac{Gmm^1(x-r)}{2r^3}$

3) $F=\frac{Gmm^1(x-r)}{r^3}$

4) $F=\frac{Gmm^1(2r-x)}{r^3}$

28. $2r<x<2R$

1) $F=\frac{Gmm^1}{4(x-r)^2}$ 2) $F=\frac{Gmm^1}{(x-r)^2}$

3) $F=\frac{Gmm^1}{(x-r)^3}$ 4) $F=\frac{2Gmm^1}{(x-r)^2}$

29. $x>2r$

1) $F=\frac{2GMm^1}{(x-R)^2}+\frac{Gmm^1}{(x+r)^2}$

2) $F=\frac{GMm^1}{2(x-R)^2}+\frac{Gmm^1}{(x+r)^2}$

3) $F=\frac{GMm^1}{(x-R)^2}+\frac{Gmm^1}{(x+r)^2}$

4) $F=\frac{GMm^1}{(x-R)^2}+\frac{Gmm^1}{(x-r)^2}$

30. A mass m extends a vertical helical spring of spring constant k by x m at the surface of earth. Extention in spring by the same mass at height of h metre above the surface of earth is

1) $\frac{GMm}{k(R+h)}$ 2) $\frac{GMm}{kR^2}$

3) $\frac{(R+h)^2}{R^2}x$ 4) $\frac{R^2}{(R+h)^2}x$

31. Four particles each of mass M are located at the vertices of a square with side L. The gravitational potential due to this at centre of square is

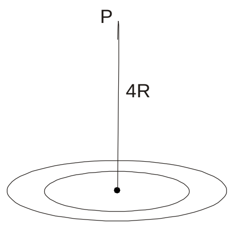
1) $-\sqrt{32}\frac{GM}{L}$ 2) $-\sqrt{64}\frac{GM}{L^2}$

3) Zero 4) $-\sqrt{16}\frac{GM}{L}$

32. The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio $3:4$. If the two shells coalesce into single one such that surface charge density remains same, then the ratio of potential at an internal point of the new shell to shell A is equal to :

1) 3 : 2 2) 4 : 3 3) 5 : 3 4) 5 : 6

33. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is



1) $\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$ 2) $-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$

3) $\frac{GM}{2R}$ 4) $\frac{2GM}{5R}\left(\sqrt{2}-1\right)$

34. The gravitational force in a region is given by, $\vec{E}=ay\hat{i}+ax\hat{j}$. The work done by gravitational force to shift a point mass m from $(0,0,0)$ to (x_0,y_0,z_0) is

1) $max_0y_0z_0$ 2) max_0y_0

3) $-max_0y_0$ 4) 0

35. Two identical thin rings each of radius 'R' are co-axially placed at a distance 'R'. If the rings have a uniform mass distribution and each has mass m_1 and m_2 respectively, then the work done in moving a mass 'm' from the centre of one ring to that of the other is:

1) zero 2) $\frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

3) $\frac{Gm\sqrt{2}(m_1+m_2)}{R}$ 4) $\frac{Gm_1m(\sqrt{2}+1)}{m_2R}$

36. The gravitaional field in a region due to a certain mass distribution is given by $\overline{E}=\left(4\overline{i}-3\overline{j}\right)N/kg$. The work done by the field in moving a particle of mass 2 kg from (2m, 1m) to $\left(\frac{2}{3}m,2m\right)$ along the line 3x+4y=10 is

1) $-\frac{25}{3}N$ 2) $-\frac{50}{3}N$

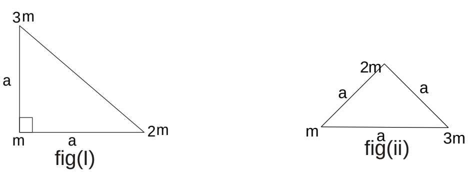
3) $\frac{25}{3}N$ 4) zero

37. A particle of mass 1kg is placed at a distance of 4m from the centre and on the axis of a uniform ring of mass 5kg and radius 3m. The work done to increase the distance of the particle from 4m to $3\sqrt{3}m$ is.

1) $\frac{G}{3}J$ 2) $\frac{G}{4}J$

3) $\frac{G}{5}J$ 4) $\frac{G}{6}J$

38. Consider two configurations in fig (i) and fig(ii)

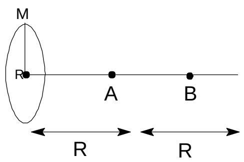


The work done by external agent in changing the configuration from fig(i) to fig(ii) is

1) Zero 2) $-\frac{6Gm^2}{a}\left(1+\frac{1}{\sqrt{2}}\right)$

3) $-\frac{6Gm^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$ 4) $-\frac{6Gm^2}{a}\left(2-\frac{1}{\sqrt{2}}\right)$

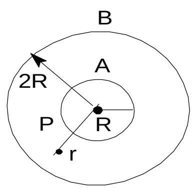
39. A ring has non-uniform distribution of mass having mass 'M' and radius 'R'. A point mass m_0 is moved from A to B along the axis of the ring. The work done by external agent against gravitational force of ring is



1) $\frac{GMm_0}{\sqrt{2}R}$ 2) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{5}}\right]$

3) $\frac{GMm_0}{R}\left[\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{2}}\right]$ 4) $\frac{GMm_0}{\sqrt{5}R}$

40. Two concentric spherical shells A and B of radii R and 2R and masses 4M and M respectively are as shown. The gravitational potential at point 'p' at distance 'r' (R <r<2R) from centre of shell is (r = 1.5R)



1) $-\frac{4GM}{R}$ 2) $-\frac{9GM}{2R}$

3) $-\frac{4GM}{3R}$ 4) $-\frac{19GM}{6R}$

KEY

LEVEL-III

1) 1	2) 1	3) 2	4) 1	5) 4
6) 1	7) 1	8) 2	9) 4	10) 3
11) 1	12) 4	13) 3	14) 4	15) 4
16) 3	17) 1	18) 3	19) 2	20) 3
21) 4	22) 1	23) 2	24) 4	25) 1
26) 3	27) 3	28) 2	29) 4	30) 4
31) 1	32) 3	33) 1	34) 2	35) 2
36) 2	37) 4	38) 3	39) 2	40) 4

HINTS

LEVEL-III

1. Gravitational force = C.P. force

$$\Rightarrow \frac{GM}{r^2}=\frac{v^2}{r'}$$

here is the radius of curvature. As per Kepler's law, time period is given by,

$$T=2\pi\sqrt{\frac{a^3}{GM}}=\frac{2\pi ab}{rv}$$

$$\left(\frac{dA}{dt}=\frac{vr}{2}\Rightarrow dt=T=\frac{2dA}{vr}=\frac{2}{rv}\pi ab\right)$$

$$v=\frac{ab}{r}\sqrt{\frac{GM}{a^3}}$$

2. Here reduced mass $=\frac{m\times nm}{m+nm}=\frac{nm^2}{m(1+n)}$

3. From conservation of energy

$$-\frac{GMm}{r_p}+\frac{1}{2}mv_p^2=\frac{-GMm}{r_a}+\frac{1}{2}mv_a^2$$

$$L=mv_pr_p$$

4. Semimajor axis=4 $\Rightarrow ae=1\Rightarrow e=\frac{1}{4}$

Semiminor axis=b

$$b=a\sqrt{1-e^2}=4\sqrt{1-\frac{1}{16}}=\sqrt{15}=\sqrt{15}$$

Required distance $=\sqrt{b^2+1}=4$

5. $r_1=(1+e)a\ ;\ r_2=(1-e)a$

$a=\frac{r_1+r_2}{2}\ ;\ r_1r_2=(1-e^2)a^2$

semilatusrectum $=\frac{b^2}{a}$

$=\frac{a^2(1-e^2)}{a}=\frac{r_1r_2}{\frac{r_1+r_2}{2}}=\frac{2r_1r_2}{r_1+r_2}$

6. Using $T=2\pi\sqrt{\frac{R^3}{GM}}=2\pi\sqrt{\frac{R^3}{G\times\frac{4}{3}\times\pi R^3D}}$

$T^2=\frac{4\pi^2R^3}{G\frac{4}{3}\pi R^3D}=\frac{3\pi}{DG}$

$\frac{3\pi}{DG}=T^2D$

7. $\frac{dA}{dt}=$ constant

$\frac{t_{AB}}{T}=\frac{(Area)SAB}{(Area)ellipse}$

$\Rightarrow\frac{\frac{\pi ab}{4}-\frac{1}{2}b(ea)}{\pi ab}$

8. On stopping, the satellite will fall along the radius r of the orbit which can be regarded as a limiting case of an ellipse with semi major axis equal to $\frac{r}{2}$

Using Keple's third law $T^2\infty r^3$

time of fall= $\frac{T'}{2}=\frac{T}{2\sqrt{8}}=\frac{\sqrt{2}T}{8}$

9. Required distance $=r_{\min}+r_{\max}$

10. Vr = constant and

$mv_1(2R)=mv_2(4R)\Rightarrow v_1=2v_2$

$\Rightarrow\frac{1}{2}mv_1^2-\frac{GMm}{2R}=\frac{1}{2}mv_2^2-\frac{GMm}{4R}$

11. $\frac{mV_1^2}{r}=\frac{GMm}{(2R)^2}$ use V_1 from previous question

12. $m\alpha\frac{PR}{\rho}$

Pressure= $h\rho g=\frac{hm(GM/R^2)}{\frac{4}{3}\pi\left[(R+h)^3-R^3\right]}$

$(h<<R)$

13. $\frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}=\frac{M_s}{\frac{4}{3}\pi S^3}$

$F=-\frac{GmM_S}{S^2}$

14. Position I. $F=\frac{Gm(m_1+m_2)}{r_1^2}$

($\therefore$ here the particle lies outside of both the shells)

Position II. $F=\frac{Gm_1}{r_2^2}$

($\therefore$ here the particle lies outside of the shell of mass m_1)

position III. Here the particle lies inside of both of the shells so $F=0$.

15. Mass of the sphere is given by $M=\frac{4}{3}\pi y^2\rho$

Gravitational force,

$F=\frac{G\left(\frac{4}{3}\pi y^3\rho\right)m}{y^2}\Rightarrow a=\frac{F}{m}$

16. Here pressure $=\frac{Gravitational\ \ pull}{area}=\frac{GMm}{r^2\times A}$

then pressure of small element da is given by

$$dp = -G\frac{\left(\frac{4\pi}{3}a^3\rho\right)(dSda\rho)}{a^2 dS}$$

Integrate $P = \frac{2\pi}{3}G\rho^2\left(R^2 - a^2\right)$

But $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

17. $d_m = D \times dl = D \times \frac{Sd\alpha}{\cos\alpha}$

Gravitational force, $dF = \frac{GMdm}{\left(\frac{S}{\cos\alpha}\right)}\cos\alpha$

total force $F = \int_{-\pi/2}^{\pi/2} \frac{MGD}{S}\cos\alpha\, d\alpha = \frac{2MGD}{S}$

18. $F_x = O = \frac{\sqrt{3}Gm^2}{12d^2}\cos 30 - \frac{Gm^2}{4d^2}\cos 60$

$F_y = \frac{\sqrt{3}Gm^2}{12d^2}\cos 60 + \sqrt{3}\frac{Gm^2}{3d^2} + \frac{Gm^2}{4d^2}\cos 30$

19. $E_1 = \frac{\rho R}{6\varepsilon_0}$, $E_c = \frac{-\rho(R/2)^3}{3\varepsilon_0 R^2}$

$E_{net} = E_1 + E_c$; $\rho = \frac{M}{\frac{4}{3}\pi R^3}$; $\varepsilon_0 = \frac{1}{4\pi G}$

20. Error $= m(g_2 - g_1) = mg\left[\frac{2h_1}{R} - \frac{2h_2}{R}\right]$

21. $\frac{Gm^2}{(2r)^2} + \frac{2Gm^2}{\left(\sqrt{2}r\right)^2}\cos 45 = \frac{mv^2}{r}$

22. $dF = \frac{GMdm}{3R^2}$; $F = \Sigma dF\cos\theta$

23. $M' = \frac{M}{8}$; $F_{net} = F_1 - F_2$

$F_{net} = \frac{GMm}{d^2} - \frac{GMm}{8\left(d - \frac{R}{2}\right)^2}$

24. $M = \frac{4}{3}\pi R^3\rho$; $m = \frac{4}{3}\pi r^3\rho = \frac{M}{64}$

$I_R = O = \overline{I} + \overline{I_P} + \overline{I_Q}$

$\overline{I_P} = -\overline{I_Q}$

25. $I_P = O$; $I_R = \frac{GM}{R^3}x = \frac{GM}{64}\times 2$

$I_Q = \frac{GM}{64}\left(\frac{1}{4^2}\right)$; $I = I_R - I_Q$

26. $r = \sqrt{6^2 + 2^2} = \sqrt{40}$

$V = V_R - V_P - V_Q = -\frac{GM}{6} - \left(\frac{-Gm}{r}\right)2$

27. Inside the solid sphere $F = \frac{Gmm'(x-r)}{r^3}$

28. Force is due to solid sphere only

$F = \frac{Gmm'}{(x-r)^2}$

29. force is due to both sphere and shell

$F_{sphere} = \frac{GMm'}{(x-r)^2}$; $F_{shell} = \frac{GMm'}{(x-R)^2}$

$F = F_{shphere} + F_{shell}$

30. Let the extension at height h be x' then

$x = \frac{GMm}{kR^2}$ $\left(\therefore F = kx \text{ or } x = \frac{F}{k}\right)$

then $\frac{x'}{x} = \frac{R^2}{k(R+h)^2}$; $x' = \frac{R^2}{(R+h)^2}x$

31. $U = -4\frac{GM}{L/\sqrt{2}} = -\sqrt{32}\frac{GM}{L}$

32. $4\pi r^2\rho = 4\pi r_1^2\rho + 4\pi r_2^2\rho \Rightarrow r^2 = r_1^2 + r_2^2$

$V = \frac{-GM}{r} = -\frac{G4\pi r^2\rho}{r}$

$V = -4\pi rG\rho \Rightarrow V \propto r$

$\Rightarrow \frac{V_1}{V_2} = \frac{r_1}{r_2} = \frac{3}{4} \Rightarrow \frac{r_1^2}{r_2^2} = \frac{9}{16}$

$r_1^2 : r_2^2 : r^2 = r_1^2 : r_2^2 : \left(r_1^2 + r_2^2\right) = 9 : 16 : (9+16)$

$\Rightarrow r_1 : r_2 : r = 3:4:5 = V_1 : V_2 : V$

33. $dm = \frac{M\left(2\pi r\right)dr}{\pi\left(16R^2 - 9R^2\right)4R}$

$dV = \frac{-G\left(dm\right)}{\sqrt{r^2 + 16R^2}}$; $V = \int_{3R}^{4R} dv$

$W = m\left[V_\infty - V\right]$

34. $W = \int \vec{F}.\vec{dr} = ma\int_0^{x_0 y_0 z_0}\left(y\hat{i} + x\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$

$= ma\int d\left(xy\right) = ma\left(xy\right)$

35. $: V_1 = \frac{-Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}$ and $V_2 = \frac{-Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$

$\Delta V = V_2 - V_1 = \frac{-Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R} + \frac{-Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R}$

$= G(m_1 - m_2)\left(\frac{1}{R} - \frac{1}{\sqrt{2}R}\right)$

Hence $W = m(\Delta V) = \frac{mG(m_1 - m_2)(\sqrt{2} - 1)}{\sqrt{2}R}$

36. $W = m\left(\vec{E}.\vec{dr}\right) = m\left(\vec{E}.\left(\vec{r_2} - \vec{r_1}\right)\right)$

37. $U_1 = -G$

$U_2 = \frac{-5G}{6}$, $W = U_2 - \left(-U_1\right)$

38. $GPE = \frac{-Gm_1m_2}{r}$

$W = GPE_2 - GPE_1$

39. $W = m\left[V_B - V_A\right]$

40. $V = \frac{-G4M}{\frac{3}{2}R} - \frac{MG}{2R}$

LEVEL-IV

1. (Note that an item of Column-I can match with more than one item of Column-II.)

Column-I	Column-II
(A) Modulus of gravitational potential at curvature centre of a thin hemispherical shell of radius R and mass M.	**P.** $\frac{GMm}{R}$
(B) Modulus of gravitational potential at curvature centre of a thin uniform wire, bent into a semicircle of radius R.	**Q.** $\frac{GM}{R}$
(C) Modulus of gravitational potential at curvature centre of a thin non-uniform wire, bent into a semicircle of radius R.	**R.** $\frac{GM}{R^2}$

1) A-Q, B-Q, C-Q 2) A-P, B-Q, C-R
3) A-R, B-P, C-Q 4) A-Q, B-P, C-P

2. Match the following
Note that an item of Column-I can match with more than one item of Column-II). If our planet suddenly shrinks in size, still remaining perfectly spherical with mass remaining unchanged.

Column-I	Column-II
A) Duration of the day	**P) increase**
B) Kinetic energy of rotation	**Q) un changed**
C) Duration of the year	**R) decrease**

The matching grid
1) A-P, B-R, C-Q 2) A-R, B-P, C-Q
3) A-R, B-Q, C-P 4) A-Q, B-P, C-R

3. Match the following
(Note that an item of Column-I can match with more than one item of Column-II.)
When a planet moves around the sun

Column-I	Column-II
(A) Its angular momentum	**P) increases**
(B) When it is near the sun its speed	**Q) constant**
(C) When it is near the sun its potential energy	**R) decreases**

1) A-P, B-R, C-Q 2) A-R, B-P, C-Q
3) A-Q, B-P, C-R 4) A-R, B-Q, C-P

4. **Match the following**
(Note that an item of Column-I can match with more than one item of Column-II). A satellite is revolving round the earth in an elliptical orbit.

Column-I	Column-II
(A) Gravitational force exerted by earth and centripetal force at some points only can be	**P) Zero**
(B) Work done by gravitational force in some small parts of orbit can be	**Q) Equal**
(C) In comparision of centripetal force at some point magnitude of gravitational force can be	**R) Greater**

1) A-P, B-R, C-Q 2) A-R, B-P, C-Q
3) A-Q, B-R, C-P 4) A-Q, B-P, C-R

5. **Match the following**
(Note that an item of Column-I can match with more than one item of Column-II). Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in same sense.Their periods of revolution are 1hr. and 8hrs. respectively. The radius of orbit of S_1 is 10^4 km.

Column-I	Column-II
A) Speed of Ist satellite	**P)** $\pi\times10^4 km/h$
(B) Speed of IInd satellite	**Q)** $3\pi\times10^4 km/h$
(C) Minimum magnitude of relative velocity between the two satellites	**R)** $2\pi\times10^4 km/h$

1) A-R, B-P, C-P 2) A-R, B-P, C-Q
3) A-Q, B-R, C-P 4) A-Q, B-P, C-R

6. **Match the Columns**

Column-I	Column - II
A) Concept of the elliptical path	**P) at the poles on surface of earth**
B) Gravitational attraction we force.	**Q) Decreases as go upwards from surface of earth**
C) Acceleration due to gravity	**R) Keppler's 1st law**
D) Acceleration due to gravity is maximum.	**S) Kepper's 2nd law**
	T) Newton's Law

1) A-R,B-P,C-T,D-S 2)A-R,B-T,C-Q,D-P
3) A-T,B-R,C-P,D-Q 4) A-Q,B-P,C-R,D-S

7. **A satellite of mass m is moving in a circular orbit of radius r = (R_e+h) around earth of radius R_e and mass M_e, and density of earth ρ. Match the following**

Column-I	Column - II
A) Orbital velocity of the satellite	**P)** $T=\sqrt{\frac{2\pi}{GM_e}}r^{3/2}$
B) Kinetic energy of the satellite.	**Q)** $\frac{GM_e m}{2r}$
C) Potential energy of the satellite	**R)** $\frac{-GM_e m}{2r}$
D) Total energy of the satellite	**S)** $\frac{-GM_e m}{r}$
E) Time period of the satellite.	**T)** $\sqrt{\frac{GM_e}{r}}$

1) A-R, B-P, C-T, D-S, E- P
2) A-R, B-T, C-Q, D-P, E- P
3) A-T, B-Q, C-S, D-R, E- P
4) A-Q, B-P, C-R, D-S, E- P

8. **If the mass of the earth is increased by 1% without change in its radius, then**
a) the value of 'g' increases by 1%
b) the value of 'g' decreases by 1%
c) the binding energy of a particle on earth increases by 1%
d) the mean density of earth increases by 1%
1) a, c & d are true 2) b, c & d are true
3) a & c are true 4) b & c are true

9. **A frame S_1 is moving uniformly with respect to an inertial frame S_2. If $\vec{a_1}$ is the acceleration of a particle with respect to S_1 and $\vec{a_2}$ is the acceleration of the same particle with respect to S_2, then**
a) $\vec{a_1}=\vec{a_2}$ **b)** $\vec{a_1}=-\vec{a_2}$
c) S_1 is inertial frame with respect to S_2.
d) S_1 is non-inertial frame with respect to S_2
1) a & c are true 2) b & d are true
3) a & d are true 4) b & c are true

10. **When a planet moves around the sun**
a) its angular momentum remains constant.
b) it moves faster when it is nearer to the sun.
c) its total energy increases when it goes nearer to the sun.
d) its potential energy decreases when it goes nearer to the sun.
1) only a & b are true
2) only b & c are true
3) only a, b & d are true
4) all are true

11. **Two spherical planets have the same mass but densities in the ratio 1:8. For these planets, the**
a) acceleration due to gravity will be in the ratio 4:1.
b) acceleration due to gravity will be in the ratio 1:4.
c) escape velocities from their surfaces will be in the ratio $\sqrt{2}:1$.
d) escape velocities from their surfaces will be in the ratio $1:\sqrt{2}$.
1) only b & d are true
2) only b & c are true
3) only a, c & d are true
4) only a & b are true

12. **For a planet moving around the sun in an elliptical orbit of semimajor and semiminer axes a and b respectively, and period T, which of the following statements are true?**
a) the torque acting on the planet about the sun is non-zero
b) The angular momentum of the planet about the sun is non-zero
c) The areal velocity is $\pi ab/T$
d) The planet moves with a constant speed around the sun
1) a, b 2) b, c
3) c, d 4) d, a

STATEMENT TYPE

In the following questions, each question contains Statement-I (Assertion) and Statement-II (Reason). Each question has four choices (1), (2), (3) and (4) out of which only one is correct. The Options are:

1) Statement-1 is true, Statement-II true; Statement-II is a correct explanation for Statement-1

2) Statement-I is true, Statement-II true; Statement-II is not a correct explanation for Statement-1

3) Statement-1 is true, Statement-II false

4) Statement-II is false, Statement-II true

13. **Statement-1: Escape velocity is independent of the angle of projection.**
Statement-2: Excape velocity from the surface of earth is $\sqrt{gR}$ where R is radius of earth.

14. **Statement-1: Work done in gravitational field in cyclic process is zero**
Statement-2: Work done in conservative field does not depend upon path.

15. **Statement-I: Gravitational potential is zero inside a shell.**
Statement-II: Gravitational potential is equal to the work done in bringing a unit mass from infinity to a point inside gravitational field.

16. **Statement-I: In its elliptical orbit around the sun, the earth is closer to the sun during summer than during winter in northern hemisphere.**
Statement-II: The angular momentum of the earth about the sun is conserved.

17. **Statement-I: A spherically symmetric shell produces no gravitational field anywhere.**
Statement-II: The field due to various mass elements cancels out, everywhere inside the shell.

18. Statement-I: For the planets orbiting around the sun, angular speed, linear speed, KE change with time, but angular momentum remains constant.
Statement-II: No torque is acting on the rotating planet, so its angular momentum is constant.

19. Statement-I: Rate of change of weight near the earth's surface with height h is proportional to h^0.
Statement-II: Since gravitational potential is given by $v = -\frac{GM}{r}$.

20. Statement-I: For a satellite revolving very near to earth's surface the time period of revolution is given by 1 h 24 min.
Statement-II: The period of revolution of a satellite depends only uopn its height above the earth's surface.

21. Statement-I: Kepler's second law can be understood by conservation of angular momentum principle.
Statement-II: Kepler's second law is related with areal velocity which can furthr be proved to be based on conservation of angular momentum as $(dA/dt) = (r^2\omega)/2$.

22. Statement-I: The force of gravitation between sphere and a rod of mass M_2 is $= (GM_1M_2)/r$.
Statement-II: Newton's law of gravitation holds correct for point masses.

23. Statement-I: Two stars start moving towards each other due to mutual force of attraction but centre of mass remains at rest.
Statement-II: External force is absent.

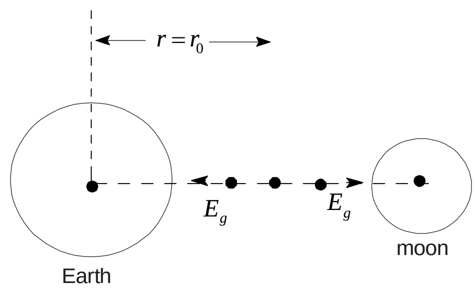
24. Statement-I: when a planet moves in elliptical orbit around sun, its angular momentum about sun remains conserved.
Statement-II: Total energy of the planet remains conserved.

25. Statement-I: Satellite is put in an orbit at a height where air resistance is present. Then orbital velocity of the satellite will decrease.
Statement-II: Due to air resistance a lot of heat will be produced which may burn satellite.

26. Statement-I: The magnitude of gravitational potential at the surface of solid sphere is less than that of the centre of sphere.
Statement-II: Due to solid sphere, gravitational potential is same within the sphere.

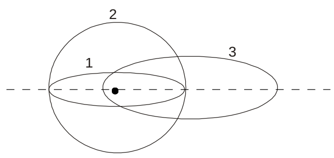
27. Statement-I: Earth does not retain hydrogen molecules and helium atoms in its atmosphere, but does retain much heavier molecules, such as oxygen and nitrogen.
Statement-II: Lighter molecules in the atmo sphere have translational speed that is greater or closer to escape speed of earth.

28. Statement-I: It takes more fuel for a spacecraft to travel from the earth to moon than for the return trip.
Statement-II: The point of zero gravitational field intensity due to earth and moon is lying nearer to moon, i.e., in diagram shown, for $r < r_0, E_g$ is towards earth's centre, and at $r < r_0, E_g$ is zero.



29. Statement-I: Consider a satellite moving in an elliptical orbit around the earth. As satellite moves, the work done by gravitational force of earth on satellite for any small part of the orbit is zero.
Statement-II: KE of the satellite in the above described case is not constant as it moves around the earth.

30. **Statement-I: Three orbits are marked as 1, 2 and 3. These three orbits have same semi-major axis although their shapes (eccentrici ties) are different. The three identical satellites are orbiting in these three orbits, respectively. These three satellites have the same binding energy.**



Statement-II: Total energy of a satellite depends on the semi-major axis of orbit according to the expression, $E = (-GMm)/(2a)$.

31. **Statement-I: Two satellite are following one another in the same circular orbit. If one satellite tries to catch another (leading one) satellite, then it can be done by increasing its speed without changing the orbit.**

Statement-II: The energy of earth-satellite system in curcular orbit is given by $E = (-GMm)/(2r)$**, where r is the radius of the circular orbit.**

32. **Statement-I: An astronaut in an orbiting space station above the earth experiences weight lessness.**

Statement-II: An object moving around the earth under the influence of earth's gravitational force is in a state of 'free fall'.

ASSERTION & REASON

In each of the following questions, a statement is given and a corresponding statement or reason is given just below it. In the statements, mark the correct answer as

1)I f both Assertion and Reason are true and Reason is correct explanation of Assertion.

2) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

3) If Assertion is true but Reason is false.

4) If both Assertion and Reason are false.

33. **Assertion: If earth suddenly stops rotating about its axis, then the value of acceleration due to gravity will become same at all the places.**

Reason: The value of acceleration due to gravity is independent of rotation of earth.

34. **Assertion: Orbital velocity of a satellite is less than its escape velocity.**

Reason: Orbit of a satellite is within the gravitational field of earth whereas escaping is beyond the gravitational field of earth.

35. **Assertion: The time period of revolution of a satellite close to surface of earth is smaller than that revolving away from surface of earth.**

Reason: The square of time period of revolution of a satellite is directly proportional to cube of its orbital radius.

36. **Assertion: Generally the path of projectile from the earth is parabolic but it is elliptical for projectiles going to a very large height.**

Reason: The path of a projectile is independent of the gravitational force of earth.

37. **Assertion: We can not move even a finger without disturbing all the stars.**

Reason: Everybody in this universe attracts every other body with a force which is inversely proportional to the square of distance between them.

38. **Statement-I: When a body is projected with velcoity $v = v_0$ (where v_0 is orbital velocity) then path of the projectile is circular.**

Statement-II: Gravitional force between body and the earth provides the centripetal force.

39. **Statement - I : For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is 4π GM.**

Statement - II : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$. It flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface

40. Statement-I: Orbiting satellite or body has K.E. of always less than that of Potential energy.

Statement-II : For any bound state, the magnitude of potential energy is always twice that of kinetic energy (K.E.)

41. Statement-I : There is almost no effect of rotation of earth at poles.

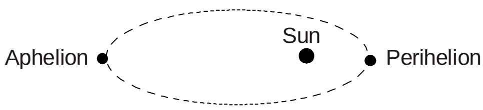
Statement-II : Because rotation of earth is about polar axis.

42. Statement-I: If the Earth suddenly contracts to 1/n th of its present size without any change in its mass. The duration of the new day will be $\frac{24}{n^2}$ hrs.

Statement-II: When no external torque acts on the system, the total angular momentum remain conserved.

For problems 43-45:

The orbit of Pluto is much more eccentric than the orbits of the other planets. That is, instead of being nearly circular, the orbit is noticeably elliptical. The point in the orbit nearest to the sun is called the perihelion and the point farthest from the sun is called the aphelion.



43. At perihelion, the gravitational potentialenergy of Pluto in its orbit has

1) its maximum value
2) its minimum value
3) The same value as at every other point in the orbit
4) value which depends on sense of rotation.

44. At perihelion, the mechanical energy of Pluto's orbit has

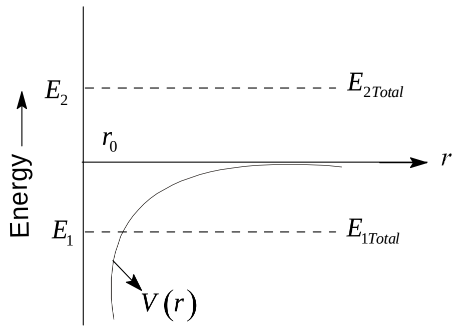
1) its maximum value
2) its minimum value
3) The same value as at every other point in the orbit
4) value which depends on sense of rotation.

45. As pluto moves from the perihelion to the aphelion, the work done by gravitational pull of Sun on Pluto is

1) is zero
2) is positive
3) is negative
4) depends on sence of rotation

For problems 46-48:

In the graph shown, the PE of earth-satellite system is shown by solid line as a function of distance r (the separation between earth's centre and satellite). The total energy of the two objects which may or may not be bounded to earth are shown in figure by dotted lines.



46. mark the correct statement(s).

1) The object having total energy E_1 is bounded one
2) The object having total energy E_2 is bounded one.
3) Both the objects are bounded.
4) Both the objects are unbounded.

47. If object having total energy E_1 is having same PE curve as shown in figure, the,

1) r_0 is the maximum distance of object from earth's centre
2) this object and earth system is bounded one
3) The KE of the object is zero when $r = r_0$
4) All the above

48. If both the objects have same PE curve as shown in figure, then

1) for objects having total energy E_2 all values of r are possible

2) for object having total energy E_2 value of $r < r_0$ are only possible

3) for object having total energy E_1 all values of r are possible

4) none of the above

PREVIOUS QUESTIONS

49. Two particles of equal mass 'm' go around a circle of radius R under the action of their mutual gravitational attaction. The speed of each particle with respect to their centre of mass is (2011)

1) $\sqrt{\frac{Gm}{3R}}$ 2) $\sqrt{\frac{Gm}{2R}}$ 3) $\sqrt{\frac{Gm}{R}}$ 4) $\sqrt{\frac{Gm}{4R}}$

50. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is (2011)

1) $-\frac{9Gm}{r}$ 2) zero 3) $-\frac{4Gm}{r}$ 4) $-\frac{6Gm}{r}$

51. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = acceleration due to gravity on the surface on the earth) in terms of R, the radius of the earth, is (2009)

1) $\frac{R}{\sqrt{2}}$ 2) $\frac{R}{2}$ 3) $\sqrt{2}R$ 4) 2R

52. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 $km\ s^{-1}$, the escape velocity from the surface of the planet would be (2008)

1) 1.1 kmS^{-1} 2) 11 kmS^{-1}
3) 110 kmS^{-1} 4) 0.11 kmS^{-1}

53. Statement-1: For a mass M kept at the centre of a cube of side 'a', the flux of gravitational field passing through its sides is 4π GM

Statement-2 : If the direction of a field due to a point source is radial and its dependence on the distance 'r' from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. (2008)

1) Statement-1 is false, Statement-2 is true

2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

3) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

4) Statement-1 is true, Statement-2 is false

54. Average density of the earth (2005)

1) does not depend upon g

2) is a complex function of g

3) is directly proportional to g

4) is inversely proportional to g

55. A double star system consists of two stars A and B which have time periods T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. (2006)

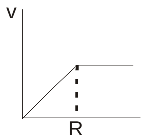
1) If $T_A > T_B$ then $R_A > R_B$

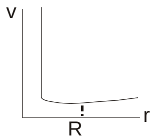
2) $T_A > T_B$ then $M_A > M_B$

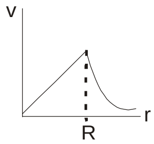
3) $\left(\frac{T_A}{T_b}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

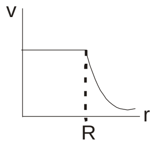
4) $T_A = T_B$

56. **A spherically symmetric gravitational system of partcles has a mass density $\rho = \begin{cases} \rho_0 & for\ r \le R \\ 0 & for\ r > R \end{cases}$ Where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as function of distance r from the centre of the system is represented by (2008)**

1) 

2) 

3) 

4) 

57. **Distance between the centre of two stars is 10a. the masses of these stars are M and $16M$ and their radii a and 2a respectively. A body of mass m is fired straight from the surface of the larger star towards the surface of the smaller star. What should be its minimum initial speed to reach the surface of the smaller star ? (2007)**

1) $\sqrt{\dfrac{GM}{a}}$ 2) $\dfrac{1}{2}\sqrt{\dfrac{5GM}{a}}$

3) $\dfrac{3}{2}\sqrt{\dfrac{GM}{a}}$ 4) $\dfrac{3\sqrt{5}}{2}\sqrt{\dfrac{GM}{a}}$

58. **There is a crater of depth $\dfrac{R}{100}$ on the surface of the moon (radius R). A Projectile is fired vertically upward from the crater with velocity, which is equal to the escape velocity v from the surface of the moon. Find the maximum height attained by the projectile. (2003)**

1) 90R 2) 95 R
3) 99.5 R 4) 50 R

59. **Gravitational acceleration on the surface of a planet is $\dfrac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be $11kms^{-1}$, the escape speed on the surface of the planet in kms^{-1} will be (2010)**

1) 3 2) 6 3) 9 4) 12

60. **A satellite is moving with a constant speed 'v' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is (2011)**

1) $\dfrac{1}{2}mv^2$ 2) mv^2

3) $\dfrac{3}{2}mv^2$ 4) $2mv^2$

KEY

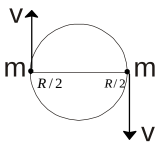
LEVEL - IV

1)1	2) 2	3) 3	4) 4	5) 1
6)2	7) 3	8)1	9)1	10)3
11)1	12)2	13)3	14)1	15)4
16)1	17)4	18)1	19)2	20)3
21)1	22)4	23)1	24)2	25)2
26)3	27)1	28)1	29)4	30)1
31)4	32)1	33)3	34)1	35)1
36)3	37)1	38)2	39)1	40)1
41)1	42)1	43)2	44)3	45)3
46)1	47)1	48)2	49)2	50)1
51)4	52)3	53)2	54)1	55)4
56)3	57)4	58)3	59)1	60)2

HINTS

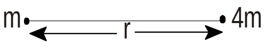
LEVEL - IV

49. $\dfrac{Gm^2}{R^2} = \dfrac{mv^2}{(R/2)}$



$$\frac{Gm^2}{R^2} = 2\frac{mv^2}{R}$$

$$v^2 = \frac{Gm}{2R};\quad v = \sqrt{\frac{Gm}{2R}}$$

50. 

Location of point from m, where field is zero

$$r_1 = \left(\frac{\sqrt{m}}{\sqrt{m}+\sqrt{4m}}\right)^r = \frac{r}{3}$$

$$r_2 = \left(\frac{\sqrt{4m}}{\sqrt{m}+\sqrt{4m}}\right)^r = \frac{2r}{3}$$

Potential at this point; $V_p = \dfrac{-Gm}{r_1} - \dfrac{G4m}{r_2}$

$$V_p = -\frac{Gm}{(r/3)} - \frac{4Gm}{(2r/3)}$$

$$= -\frac{3Gm}{r} - \frac{6Gm}{r} = \frac{9Gm}{r}$$

51. Acceleration due to gravity at a height h above the surface of the earth is expressed as

$$g' = g\left(\frac{R}{R+h}\right)^2$$

According to the question $g' = \dfrac{g}{9}$

Replacing it in the equation of g', we get

$$\frac{g}{9} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{9} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \frac{1}{3} = \frac{R}{R+h} \Rightarrow R+h = 3R \Rightarrow h = 2R$$

52. The expression of escape velocity is

$$v_e = \sqrt{\frac{2GM}{R}}$$

According to question,

$$M_{planet} = 10\,M_{Earth};\qquad R_{Planet} = \frac{R_{Earth}}{10}$$

$$v_p = \sqrt{\frac{2G10(M_{Earth})}{R_{Earth}/10}} = \sqrt{100\left[\frac{2GM_E}{R_E}\right]}$$

$$v_P = 10\sqrt{\frac{2GM_E}{R_E}} = 10v_{Escape\,earth}$$

$$v_P = 10\times 11 = 110 km/s$$

53. Statement-I: The gravitational flux associated with a closed loop.

$$\int \vec{E}.\vec{d}s = \frac{GM}{r^2}4\pi r^2$$

$$\int \vec{E}.\vec{d}s = (4\pi G)(M_{enclosed})$$

True

Statement : 2 : The gravitational flux with a closed surface only depends on mass enclosed and not on shape or size of closed surface.

True and also it is correct explanaton of statement-1.

54. Value of g depends upon ρ. ρ does not depend on g.

55. Conceptual

56. For $r \le R$

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad \to (1)$$

here, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

substituting in Eq(1) we get $v \propto r$

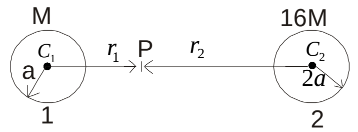
i.e., $v-r$ graph is a straight line passing through origin.

for $r > R$

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{3}{4}\pi R^3\right)\rho_0}{r^2} \text{ or } v \propto \frac{1}{\sqrt{r}}$$

The corresponding $v - r$ graph will be as shown in option (3)

57 Let there are two stars 1 and 2 as shown be low.



Let P is a point between C_1 and C_2, where gravitational field strength is zero. Hence

$$\frac{GM}{r_1^2} = \frac{G(16M)}{r_2^2};\ \frac{r_2}{r_1} = 4,\quad r_1 + r_2 = 10a$$

$$\therefore\ r_2 = \left(\frac{4}{4+1}\right)(10a) = 8a$$

$r_1 = 2a$

Now, the body of mass m is projected from the surface of large star towards the smaller one. Between C_2 and P it is attracted towards 2 and between C_1 and P it will be attracted towards 1. Therefore, the body should be projected to just cross point P because beyond that the particle is attracted towards the smaller star itself.

From conservation of mechanical energy $\frac{1}{2}mv^2$

= potential energy of the body at P

– potential energy at the surface of larger star.

$$\therefore\ \frac{1}{2}mv_{\min}^2 = \left[\frac{GMm}{r_1} - \frac{16GMm}{r_2}\right]$$

$$-\left[-\frac{GMm}{10a-2a} - \frac{16GMm}{2a}\right]$$

$$\frac{1}{2}mv_{\min}^2 = \left(\frac{45}{8}\right)\frac{GMm}{a}$$

$$v_{\min} = \frac{3\sqrt{5}}{2}\left(\sqrt{\frac{GM}{a}}\right)$$

58. Speed of particle at A, v_A,

= escape velocity on the surface of earth

$$= \sqrt{\frac{2GM}{R}}$$

At highest point B, $v_B = 0$

Applying conservation of mechanical energy, decrease in kinetic energy

= increase in gravitational potential energy

$$= \frac{1}{2}mv_A^2 = U_B - U_A = m(v_B - v_A)$$

$$\frac{v_A^2}{2} = v_B - v_A$$

$$\frac{GM}{R} = -\frac{GM}{R+h} - \left[\frac{-GM}{R^3}\left(1.5R^2\right) - 0.5\left(R - \frac{R}{100}\right)^2\right]$$

$$\frac{1}{R} = -\frac{1}{R+h} + \frac{3}{2R} - \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^2 . \frac{1}{R}$$

Solving this equation, we get $h = 99.5R$

59. $$g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$

$$g \propto \rho R;\quad R \propto \frac{g}{\rho}$$

Now excape velocity, $v_e = \sqrt{2gR}$

$$v_e \propto \sqrt{gR}$$

$$v_e \propto \sqrt{g \times \frac{g}{\rho}} \propto \sqrt{\frac{g^2}{\rho}}$$

$$(v_e)_{planet} = \left(11kms^{-1}\right)\sqrt{\frac{6}{121} \times \frac{3}{2}} = 3kms^{-1}$$

60. In circular orbit of a satellite, potential energy

$$= -2 \times (kinetic\ energy) = -2 \times \frac{1}{2}mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$

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